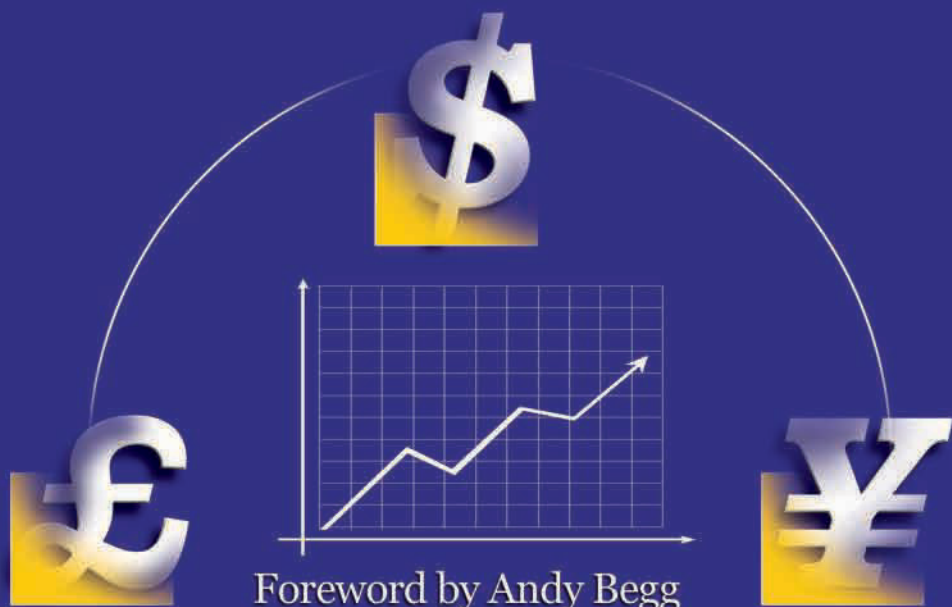


Sergiy Klymchuk

MONEY PUZZLES

*On Critical Thinking and
Financial Literacy*



Sergiy Klymchuk

MONEY PUZZLES

On Critical Thinking and Financial Literacy

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Preface

One more book on puzzles. Why? What is new? What is special?

We live in the age of breakthroughs in revolutionary changes in knowledge and information. Critical thinking and problem solving skills are crucial for success and confidence in a society based on a competitive knowledge economy. The amount of information is doubling every several years. We have to select, comprehend and analyse new information very fast.

This book is about analysing basic financial information, effective critical thinking, problem solving, and financial literacy. It is a collection of 120 tricky practical mathematical puzzles dealing with money, a topic of universal appeal. The mathematics in the book is not about calculations. They are easy: the readers need to know only the four basic arithmetic operations with whole numbers. The emphasis is on different ways of thinking, in particular, lateral thinking - going beyond the boundaries, strategic thinking for decision making and also mathematical, commercial, versatile and other ways of thinking.

The book is for everyone from 12-years old to 100-years old. The book can motivate teenagers to look at mathematics from another angle, be an excellent brain gym for professional managers, and be the perfect way to keep the mind sharp for people in their golden years. The puzzles are designed to both entertain and educate. Solving the

puzzles can enhance a person's brain power and mental fitness and be an investment in their intellectual capital. It can be a good resource for school teachers to motivate their students and stimulate their interest in the subject in a pleasurable and exiting way. It can be also an enjoyable family activity. The readers can strengthen their effective thinking skills and have fun with simple interesting mathematics using money, a context that is based on real-life experiences, easy to understand and related to everyone.

Most of the puzzles look simple until you try to solve them. Some of them are easy but the majority are quite tricky, challenging and even teasing. Very often the readers will be surprised to see unexpected answers and solutions. I do anticipate that everyone will find some puzzles that make them ask "How come?" when they look at the 'Answers Only' section. This is one feature of the book that makes it special. Apart from answers and solutions the readers will find many comments on common misconceptions, misinterpretations and misleading ways of thinking. Many of the problems can be done in several ways and although the solutions often only indicate one method, I expect that many readers will come up with different approaches.

The puzzles in the book are from a variety of different sources. Some were picked up from conversations with colleagues and friends. Some were created or converted

into a money context by the author. Most of the time it is impossible to identify the author of a puzzle as one can encounter many different versions of it in different books printed in different languages. Often the authors of famous old puzzles are unknown. These puzzles are like fairy tales.

A couple of words to those who feel anxious towards maths. In some cultures it is socially acceptable to admit being bad at maths. It is perfectly okay to say: "I never could understand maths". But it is unlikely you will hear: "I never could count money". For those who have a hang-up towards maths: this book is more about counting money than doing maths problems. It is about financial literacy in a real practical context. You will see that precise thinking is needed inside and outside mathematics, especially when you count money!

I believe this book will help people to think more effectively, not just about mathematics or money but about other areas of life. The skills that can be gained from the book are very practical and will benefit everyone, no matter what they are doing. In particular, those skills can give the readers confidence in handling financial matters and help them to see business opportunities. This can lead to an enhancement of their entrepreneurial skills and an ability to think about creating their jobs by themselves rather than relying on the government or established enterprises. This is very important in a highly competitive economic environment.

Open your mind, dear reader, boost your performance, stretch your effectiveness and have fun!

Good luck!

Sergiy Klymchuk

Acknowledgments

I sincerely believe that this book will be a good way to popularise critical thinking, financial literacy and a mathematical way of thinking, and help in promoting and raising the profile of mathematics education in New Zealand schools and the wider community, to the long term benefit of the country.

I am very grateful to the following sponsors and supporters who kindly assisted with printing and distributing complimentary copies of the book to all New Zealand intermediate and high schools, tertiary institutions, major libraries and community centres and creating a website for discussion forum and readers' feedback:

ASB Bank - the major sponsor (www.asbbank.co.nz)

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Special thanks to Andy Begg and Peter Watson who proofread the text of the book, checked the solutions and provided some constructive comments. However, for any inaccuracies I as the author accept full responsibility.

Sergiy Klymchuk

Foreword

In this book my colleague Sergiy Klymchuk has presented a number of interesting "Money Puzzles". These link the recreational aspect of mathematics with a familiar and important context. They provide some thought provoking challenges for the reader. While they focus on a number of mathematical concepts they also show the importance of language and lateral thinking.

In puzzles and games (games being puzzles for more than one person), mathematical principles often underpin the solutions. However, because of the 'fun' context of puzzles one finds oneself doing mathematics without thinking of doing mathematics. This reflects how many people do mathematics in the real world when they are counting, buying and selling, explaining, measuring, locating, and playing and these activities are common to people of all cultures and of all ages.

Mathematics in the academic world is a collaborative activity. The context of puzzles provides an opportunity for collaboration as groups of friends, for family members, and for classmates to have fun, compete with each other, and help each other solve the puzzles. In doing this the friends, family members, and classmates will be working as mathematicians do.

When the word mathematics is mentioned, most people think of school. Very few think of it as providing another perspective to make sense of one's world. This is partly due to the way that people experienced schooling. In other cultures mathematics is viewed differently, in some it is seen as more important than language, in others it is mainly a way of getting solutions to problems, and in others it has a recreational aspect.

My hope is that this book helps people rediscover the enjoyment of both puzzles and mathematics as a leisure activity. I am sure that the puzzles will provide challenges, and I hope that they will lead readers/puzzlers to look for other collections of puzzles with different contexts to continue their exploration of one aspect of mathematics.

Dr Andy Begg

*Associate Professor
Auckland University of Technology
New Zealand*

CHAPTER 1

Buying, Selling, Earning and Paying

1



A man bought a product for \$6 and sold it for \$7. Then he bought the **same** product for \$8 and sold it for \$9.

?

What is his total profit?

2



Two girls Helen and Jenny decided to buy a post-card. Helen needed 70c more to buy the card, and Jenny needed 10c more to buy the card. When they added their money, they still did not have enough money to buy the card.

?

How much did the card cost? (There are only 5,10,20, and 50 cent coins in the country).

Buying, Selling, Earning and Paying

3



A consultant worked for one company for \$60 per hour. Later she worked for a second company for \$40 per hour. She earned the same amount of money in both jobs.

Was her average hourly rate for those two jobs:

- ◆ less than \$50 per hour
- ◆ exactly \$50 per hour
- ◆ more than \$50 per hour?

?

4



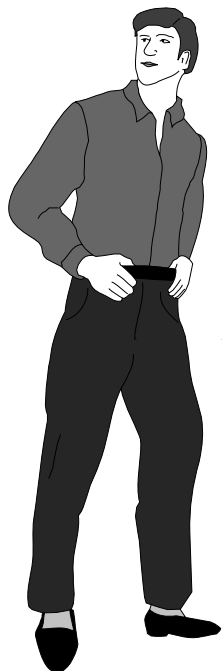
While shopping 3 women spent \$300 in 3 hours.

?

How many hours do 6 women need to spend \$600 buying with the same propensity?

Buying, Selling, Earning and Paying

5



One day a man went into a clothing shop and asked for a T-shirt. He chose one that cost \$15. He gave a \$50 note to the saleswoman. The saleswoman did not have enough change, so she went to the sport shop next door to change the \$50 note. Then she returned to her shop, gave the man the T-shirt and \$35 change. Five minutes later the owner of the sport shop came into the clothing shop and said that the \$50 note was a fake! The unfortunate saleswoman of the clothing shop carefully checked the note, agreed, and gave the sport shop owner a real \$50 note.

How much did the clothing shop lose?



Buying, Selling, Earning and Paying

6



Two friends John and Nick start working for the same salary. John receives his pay twice a month, and Nick once a month. Apart from their salary they receive regularly pay increments due to inflation starting from the second payment. The increments for John start at \$10 and increase by \$10 every time. The increments for Nick start at \$40 and increase by \$40 every time.



Who earns more?

7



Carol bought for a party several bottles of juice at \$8 per bottle and several cakes at \$11 each. The total cost was \$100.



How many cakes did she buy?

Buying, Selling, Earning and Paying

8

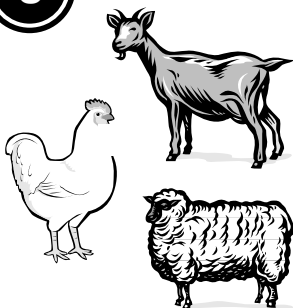


There were some qualified builders and some unqualified builders working on a certain day. A qualified builder earns \$140 a day, while an unqualified one earns \$80 a day. Together they earned \$960 on that day.

How many qualified builders were there on that day?

?

9



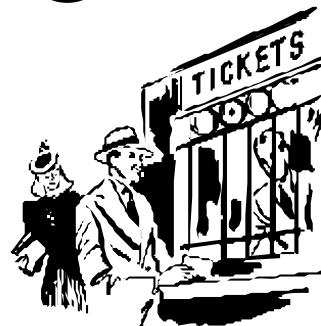
A farmer buys 100 live animals - chickens, goats and sheep - for \$1000.

How many of each does he buy if chickens cost \$5 each, goats cost \$35 each, and sheep cost \$100 each?

?

Buying, Selling, Earning and Paying

10

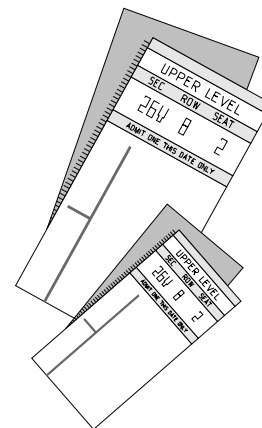


There were 2 types of tickets for a show - adult and child. An adult ticket cost \$5. A child ticket cost \$2.50. The takings were \$734.

Why was that wrong?

?

11



A man wanted to go to a show. The ticket cost \$80. The man only had a golden coin worth \$50. He went to the pawnshop and got \$40 cash and a pawn ticket for the coin. Then he met a friend and sold him the pawn ticket for \$40. With \$40 from the pawnshop and \$40 from the friend he bought the ticket to the show.

Who lost money and how much?

?

Buying, Selling, Earning and Paying

12

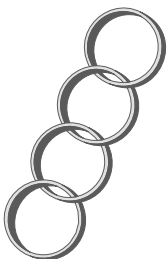


John worked for a number of days. Each day he earned the same amount of dollars as the total number of days he worked. Peter earned one dollar more per day than John, but worked one day less than John.

?

Who of them earned more money?

13



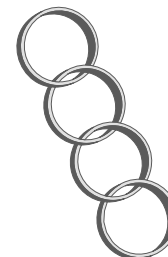
The diameter of a gold wire is 1 mm. A 100-link chain is made from it. The outer diameter of a circular link is 6 mm.

?

How much will this chain cost if such chain sells at \$10/cm?

Buying, Selling, Earning and Paying

14



The diameter of a gold wire is 2mm. The outer diameter of a circular link is 5mm.

?

How much will a 50-link chain cost if such chain sells at \$10/cm?

15



A traveller did not have the money to pay for accommodation. However he had a golden chain with 6 links. The owner of the hotel agreed to take one link per night for accommodation but did not want to take more than one cut link.

?

How can the traveller cut the chain to be able to pay for accommodation every day over 6 days?

Buying, Selling, Earning and Paying

16

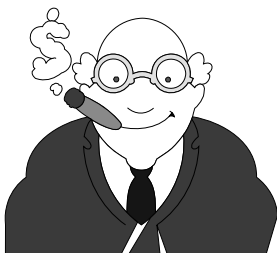


A farmer has 5 pieces of chain of three links each. He wants to connect them into one 15-link chain. The blacksmith charges \$5 for cutting a link and \$5 for welding it again.

?

What is the cheapest cost to make one 15-link chain?

17

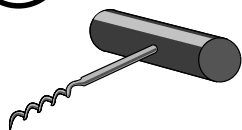


A man is buying items.
"How much would 5 cost?"
"Three dollars."
"How much would 25 cost?"
"Six dollars."
"How much would 125 cost?"
"Nine dollars."

?

What might he be buying?

18



A special gift bottle and a cork cost \$10. The bottle costs \$9 more than the cork.

?

How much does the cork cost?

Buying, Selling, Earning and Paying

19



The ticket price for the show was \$150. After reducing the price of the ticket the number of customers has tripled and the takings have doubled.

?

By how much has the price of the ticket been reduced?

20



Three travellers paid \$30 for accommodation: \$10 each. The owner of the inn decided to return \$5 to them and gave the \$5 to his servant. Since 5 is not divisible by 3, the servant returned \$3 to the travellers and kept \$2. Then he asked himself: "The three travellers paid $3 \times \$9 = \27 , and I kept \$2. $\$27 + \$2 = \$29$. But the initial payment was \$30.

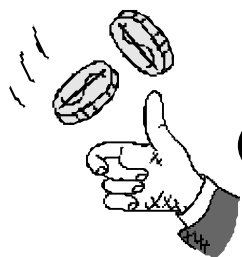
?

Where is the missing dollar?

CHAPTER 2

Comparing and Dividing Money

21



Two coins total 15 cents. One of them is **not** a 5-cent coin.

?

**What are these coins?
(There are only 5, 10, 20, and 50 cent coins in the country).**

22



Three 100 dollar notes from USA, Australia and New Zealand are randomly given to an American, an Australian and a New Zealander.

?

What are the chances that exactly two of them receive their own currency?

23

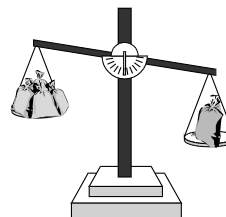


If I gave \$1 to my sister she would have twice as much as me. If she gave me \$1 we would have the same amount.

?

How much does each of us have?

24



Which is worth more: 2kg of \$1 golden coins or 1kg of \$2 golden coins?

?

25



Three backpackers cooked rice for dinner. The first gave 400g of rice and the second 200g of rice. The third backpacker did not have any rice so he gave \$6 to the other two.

?

How should they divide the \$6 between them in a fair way?

Comparing and Dividing Money

26



High in the mountains three mountaineers needed a fire to make hot drinks. They built a fire. The first contributed three pieces of wood and the second five. The third person did not have any pieces of wood so he gave \$8 to the other two.



How should they divide the \$8 in a fair way?

27



A group of 60 tourists arrived in Amsterdam. 12 of them had neither Euro nor American dollars, 35 had Euro, and 40 had American dollars.



How many tourists had both Euro and American dollars?

Comparing and Dividing Money

28



Among a group of 20 tourists 12 had American dollars, 16 had British pounds, 6 had both.



How many of them had neither American dollars nor British pounds?

29



A group of 40 exchange students arrived in Australia. 30 of them had American dollars, 20 of them had British pounds.



a) What was the smallest possible number of students who had both currencies?



b) What was the largest possible number of students who had both currencies?

Comparing and Dividing Money

30



Two fathers and two sons divided \$300 in such a way that each of them got \$100.

?

How could this happen?

31



One father gave his son \$1000, another father also gave his son \$1000. But both sons had only increased their combined money by \$1000.

?

How could this happen?

32



Two people won \$200 in a casino.

?

How much will four people win?

Comparing and Dividing Money

33



Four people divided some money in the following way. The first person took a half of the money and \$5. The second took a half of what was left and \$5. The third took a half of what was left and \$5. The fourth took a half of what was left and the last \$5.

?

What was the initial amount of money?

34



One day two workers Steve and Garry were supposed to make an equal number of golden coins. Steve came to work earlier to help Garry. He made 50 coins for Garry. After that he began to do his own job. Garry finished his job earlier than Steve and decided to help Steve too. He returned his 'debt' to Steve by making 50 coins for him and after that he made another 50 coins so that Steve had met his production requirements.

?

How many more coins did Garry make than Steve?

CHAPTER 3

Coins in Wallets, Boxes and on Tables

35



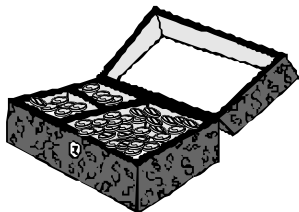
There are 10 coins in a wallet. It is known that:

- at least one of them is real;
- at least one of any two of them is fake.



How many fake coins are there in the wallet?

36



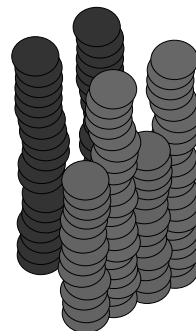
There are 10 New Zealand and 10 Australian coins of similar size and weight in a box. You take the coins out of the box without looking at them.



What is the smallest number of coins you need to take out of the box to be absolutely sure that you have 3 coins of the same country, either New Zealand or Australian?

Coins in Wallets, Boxes and on Tables

37



There are 50 coins of similar size and weight from different countries in a box:

20 American
15 Australian
10 New Zealand
5 United Kingdom

You are taking the coins out of the box without looking at them.



What is the smallest number of the coins you need to take out of the box to be absolutely sure that you have 10 coins of the same country?

38



There are 30 coins of similar size and weight from different countries in a box:

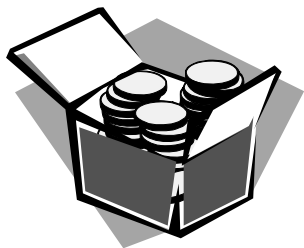
15 American, 10 Australian,
5 New Zealand. You take the coins out of the box without looking at them.



What is the smallest number of the coins you need to take out of the box to be absolutely sure that you have at least 2 New Zealand coins?

Coins in Wallets, Boxes and on Tables

39



There are 10 coins in a box. Four of them are fake, eight of them are \$1 coins.

?

a) What is the smallest possible number of fake \$1 coins that could be in the box?

?

b) What is the biggest possible number of fake \$1 coins that could be in the box?

40



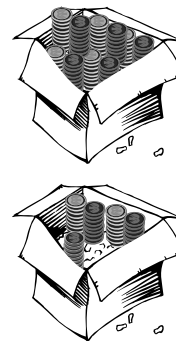
There are 5 coins in the wallet.

How would you divide them between 5 people in such a way that everyone receives one coin and one coin remains in the wallet?

?

Coins in Wallets, Boxes and on Tables

41



There are two boxes full of small coins of the same size: the first contains 10 scoops of golden coins, whereas the second contains 5 scoops of silver coins. From the first box you take one scoop full of golden coins, put them into the second box and thoroughly mix them. Then you take one scoop of that mixture and put it back to the first box.

?

Where is the larger number of "not original" coins - silver coins in the first box or golden coins in the second box?

42



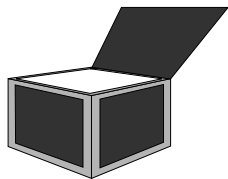
How can you put 45 coins into 9 wallets in such a way that all wallets have a different number of coins?

?

Is there only one way of doing this?

Coins in Wallets, Boxes and on Tables

43



Once upon a time an old man decided to give his money to his 3 daughters. He had 21 boxes: 7 full of golden coins, 7 half full of golden coins, and 7 empty.

How could he divide the boxes among his 3 daughters in such a way that each daughter received 7 boxes and the same amount of golden coins?

?

44

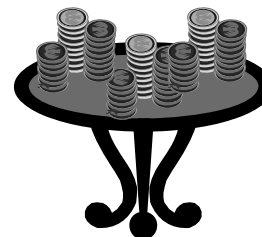


How many coins of the same size can be arranged around another of them so that each coin touches the coin in the centre and the one on each side?

?

Coins in Wallets, Boxes and on Tables

45

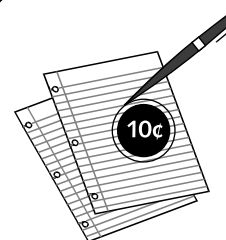


There were several coins on a table. One in front, two in the back. One in the back, two in front. One between two and three in a row.

What is the smallest number of coins that could be on the table?

?

46



On a sheet of paper draw a circle around a 10c coin. Cut the circle out.

Can you put a 20c coin through the hole?

?

47



Find 2 ways to construct a square on a table using 8 identical coins.

Coins in Wallets, Boxes and on Tables

48



Find 2 ways to construct a symmetrical cross on a table, which has equal arms, using 10 identical coins.

?

49



Find 2 ways to arrange 4 identical coins so that each coin touches the other three.

?

50



It is easy to arrange 3 identical coins to be equidistant from each other.

Can you arrange 4 identical coins to be equidistant from each other?

?

Coins in Wallets, Boxes and on Tables

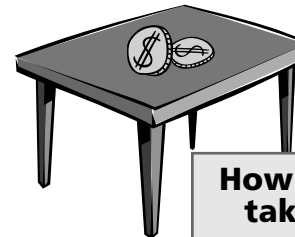
51



How many different squares are formed by the following 25 coins?

?

52



Take 2 identical coins. Put one of them flat on a table. Roll the other around the first coin so it keeps touching it.

How many revolutions does it take for the second coin to complete one circle around the first coin?

?

53



Put a small coin into an empty wine bottle. Close the bottle with a cork.

Can you remove the coin without breaking the bottle or pulling out the cork?

?

Coins in Wallets, Boxes and on Tables

54



?

Estimate how many 5-cent coins you can put one by one into an empty can of Coke.

55



There are 5 small identical coins and one large coin. The diameter of a large coin is twice as big as the diameter of a small coin.

Can you completely cover the large coin using the 5 small coins?

?

Coins in Wallets, Boxes and on Tables

56

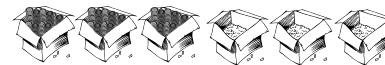
There are five cells in a row on a table. There are two coins heads up in the first two cells. The third cell is empty. There are two coins tails up in the last two cells. You need to swap the H and T coins using the following rule: a coin can be moved either by sliding it into an empty cell or by jumping over one other coin into an empty cell in any direction.



Can it be done in just 8 steps?

?

57



There are six boxes in a row. The first three are all full of coins, the next three are all empty.

How can you have them alternating by moving just one box?

?

CHAPTER 4

Operations with Money

58



How much is 2 dollars plus 2 dollars times 2 ?

59

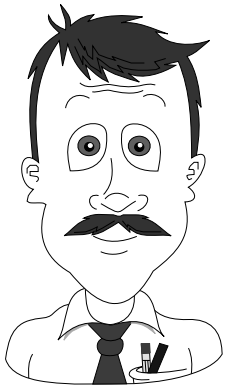
How much is a twice of a half of 8 dollars and 25 cents ?

60

How many times will you write 9 in the sequence: \$1, \$2, \$3,..., \$100 ?

?

61



Find the mistake

Step 1: obviously true equality
 $\$2 = 200c$

Step 2: square both sides
 $(\$2)^2 = (200c)^2$

Step 3: doing calculations
 $\$4 = 40,000c$

Step 4: converting cents into dollars
 $\$4 = \400

?

Operations with Money

62



?

Divide \$25 into two parts in such a way that one of them is 49 times more than the other.

63



?

Jane and Chris have the same amount of money.

How much should Chris give to Jane to have \$10 less than Jane?

64



?

The amount of 9 thousand 9 hundred 9 dollars is written as \$9909. Quickly write down the amount of 12 thousand 12 hundred 12 dollars.

Operations with Money

65



Two people are independently asked to say any amount of money in dollars expressed as a natural number. If they choose the same number they both receive valuable prizes.

?

Which number would you choose if you were asked to play this game?

66

There were 20 businessmen at a meeting each representing a different country. At the end they exchanged a coin from their home country with each other.

?

How many exchanges were there?



Operations with Money

67

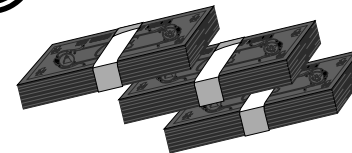


You are playing the following game with a partner. You begin with putting any amount of money in dollars as a whole number not greater than \$10. The other player adds any amount of money in dollars as a whole number not greater than \$10. Then you have your next turn, and so on. Every time each player knows how much the other player adds. The winner is the player who can first reach exactly \$100.

?

How should you play to win?

68



?

How many ways are there to change a \$50 note using \$5 and \$10 notes?

CHAPTER 5

Share Market

69

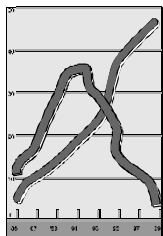


The price of a share over a certain period was increasing by 20c in the morning and decreasing by 15c in the afternoon during every working day. In the beginning of that period the price of the share was \$3.20.

How many working days did it take for the price to reach \$3.70?

?

70



There are two shares: the first is cheaper than the second. The first share is increasing in price at the rate of 5 cents a day, the second is decreasing at 10 cents a day. One day the prices became the same.

What was the difference between the prices 2 days before they became the same?

?

Share Market

71



My brother has 3 times as many shares as me. Our father has 3 times as many shares as my brother. Our grandfather has 3 times as many shares as our father. Together we have 400 shares.

How many shares do I have?

?

72



A broker did 10 less share market transactions today than yesterday. At the end of a day she makes a preliminary analysis of each transaction as being either a potential success or failure. Today she thought she had achieved 3 more successes than yesterday.

How has the number of failures changed compared to yesterday?

?

CHAPTER 6

Money and Percentages

73



The price of a product was reduced by the same number in dollars as in %.



What was the initial price?

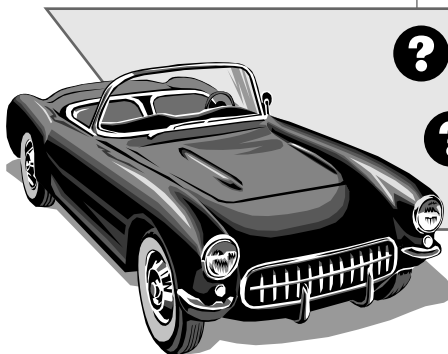
74

You are buying a car. A dealer offers you two options to choose from:

he gives you a 15% reduction from the price of the car and then charges a 10% commission

OR:

first he charges a 10% commission and then makes a 15% reduction.



a) Which option is better for you?



b) Which option is better for the dealer?

75



You are buying a computer. A salesman offers you:

- a 10% reduction then adds a 10% commission;

or

- a 20% reduction then adds a 20% commission;



a) Which option is better for you?



b) Which option is better for the salesman?

76



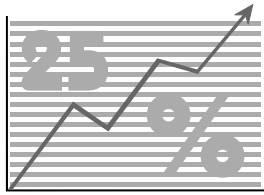
The price of a product was raised by 10%. Later it fell by 10%.



Did it become cheaper or more expensive than its initial price?

Money and Percentages

77



The price of a product increased by 25%.

?

By how many % must the new price be decreased to get the initial price?

78

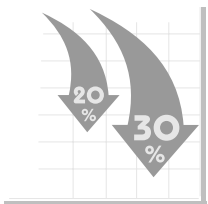


John inherited from his uncle 25% more money than his sister Helen. He wants to give her part of his money to have the same amount as her.

?

What % of his money should John give to Helen?

79



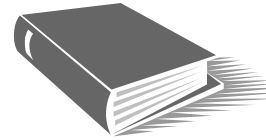
Which is the bigger reduction:

- a) a 30% cut?
- or
- b) a 20% cut followed by a 10% cut?

?

Money and Percentages

80

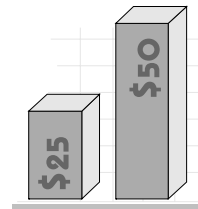


A girl paid for a book \$6 plus 25% of its price.

?

What was the price of the book?

81

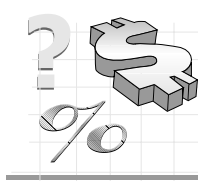


\$25 is less than \$50 by 50%.

?

By how many % is \$50 greater than \$25?

82



a) **By how many % is \$40 less than \$50?**

b) **By how many % is \$50 greater than \$40?**

?

Money and Percentages

83



After the first year of working Irene got a 10% salary increase. Since then she got another salary increase of 20%.



By how many % was her salary increased compared to the starting salary?

84



I bought 2 books. The second book is dearer than the first by 150%.



By how many % is the first book cheaper than the second?

85



The price of a product increased by 100% and then decreased by 50%.



Did the product become cheaper or dearer?

Money and Percentages

86



John's hourly wage is 40% of Peter's hourly wage.



How many % of John's hourly wage is Peter's hourly wage?

87



Murray has twice as much money as Andrew has.



By how many % is it greater?

88



Jack has 5 times as much money as Bill has.



By how many % is it greater?

Money and Percentages

89

5% → 7%

Interest rates have increased from 5% to 7%.

?

What is the percentage increase in the interest rates?

90



John received a 20% pay increase. At the same time prices for all goods and services were reduced by 20%. John spends all his salary on goods and services (no savings or investments).

?

By how many % did the purchasing power of John's salary increase?

Money and Percentages

91



A man bought a product with a 20% discount from the marked price and sold it at the market price.

?

What was his profit in % to the price he paid for the product?

92



A farmer sells 100kg of mushrooms for \$1 per kg. The mushrooms contain 99% moisture. A buyer makes an offer to buy these mushrooms a week later for the same price. However a week later the mushrooms would have dried out to 98% of moisture content.

?

How much will the farmer lose if he accepts the offer?

93



Two girls Lucy and Jennifer are working in the office. For each document she prepares Lucy is paid 10% less than Jennifer but she prepares 10% more documents per day than Jennifer.

Who earns more per day?



94



Mary had 2 separate vouchers that entitled her to 2 discounts: one voucher for 10%, the other for 20%.

What was her total discount if she used both vouchers for a single purchase?



95



The shop owner got 20% profit after selling a dress with a 20% reduction.

What was his intended % profit on the dress before the reduction?



CHAPTER 7 Magic Money

96



A lucky businessman was able to double his money every week. It took him 16 weeks to reach one million dollars.

How long did it take him to reach half a million dollars?



Magic Money

97



You put one magic \$1 coin into a box. The coin is converted into 2 such magic \$1 coins in one minute. Then those two coins are converted into 4 such coins in one minute, and so on. It takes 12 minutes to fill the box.

? How long does it take to fill the box if you put in initially 2 such magic coins instead of one?

98



Imagine a magic rain of one-dollar coins falling from the sky at a rate of one coin per second.

? How many coins will have fallen over one minute?

Magic Money

99

A magician offered a man to double his money every time he crossed a bridge. The man was so happy to accept the offer that he did not pay any attention to the fact that the magician charged \$40 for each crossing. The man crossed the bridge, doubled his money, paid \$40 to the magician, then crossed the bridge again, and so on. When he crossed the bridge for the third time, he discovered that he has only \$40 that he had to give to the magician.

? How much did the man have before he met the magician?



CHAPTER 8

Weighing Coins

100



One out of 9 similar looking coins is fake. It is lighter than a real coin.

?

How can you identify the fake coin using a balance scale and only 2 weighings?

101



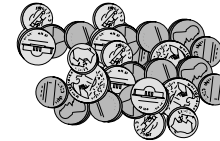
There are 27 coins. One of them is fake and lighter than a real coin.

?

How can you identify the fake coin using the balance scale and only 3 weighings?

Weighing Coins

102

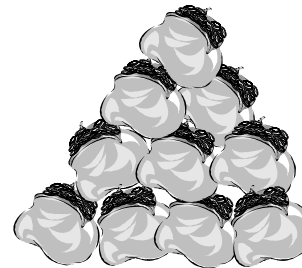


There are 81 coins. One of them is fake and lighter than a real coin.

?

How can you identify the fake coin using a balance scale and only 4 weighings?

103



There are 10 wallets each containing 10 coins. All the coins look the same. In 9 of the wallets all the coins are real and weigh 10g each. In one wallet all the coins are fake and weigh 11g each.

?

How can you identify the wallet with the fake coins using just one weighing on a scale?

CHAPTER 9

Money and Logic

Money and Logic

104



The statement is:
John buys Telecom shares *if* he receives a bonus at work.

- In what situation is the statement broken?
- What could we conclude from the above statement about John's bonus when he buys Telecom shares?
- What could we conclude from the above statement about John when he doesn't receive a bonus?



105

The statement is:
John buys Telecom shares *if and only if* he receives a bonus at work.

- In what situation is the statement broken?
- What could we conclude from the above statement about John's bonus when he buys Telecom shares?
- What could we conclude from the above statement about John when he doesn't receive a bonus?



106



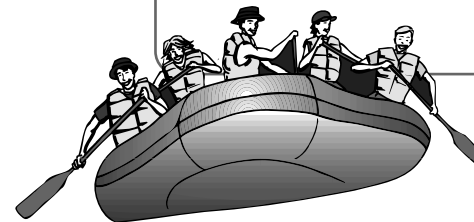
The statement is:
John buys Telecom shares *only if* he receives bonus at work or doesn't receive it.

Which word in the statement is absolutely necessary?



107

A group of refugees arrived in the country. The leader of the group was fluent in the official language of the country. The local officials gave him the following instruction: open a bank account only for those people from his group who cannot open it themselves. The leader was puzzled a bit about what he should do with himself. If he can open his bank account he shouldn't do it according to the instruction. If he cannot open it he should do it according to the instruction.



What should he do?



108

You have just **30 seconds** to solve the following puzzle after you have read it (this is important information).

There are 3 businessmen of different nationalities at the meeting: the American, Australian and New Zealander. One of them has a business worth 10 mln. USD, another 20 mln.USD and the third 30 mln. USD. One of them deals with investment, another with trade and the third with manufacturing. Each of them has their favourite drink. The American is not the richest. The person who likes beer deals with investment. The Australian doesn't like wine. The person who deals with trade is not an American. The person with 20 mln. USD business is an Australian. The New Zealander deals with manufacturing.

Who drinks whisky?



CHAPTER 10

Miscellaneous

109



Why do 1995 dollars cost more than 1990 dollars?

?

110



You found a coin with the date 369BC on it.

What can you say about that?

?

111

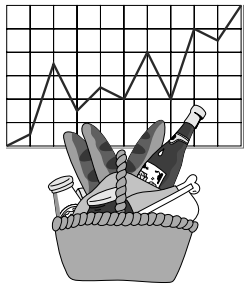


A jeweller needs to cut a golden wire 30-cm length into 30 pieces 1 cm each. It takes him 2 seconds to make one cut.

How long does it take him to do the job without a break?

?

112



Explain the following famous paradox. When the price for a cheap but essential product (e.g. bread, rice, potato) goes up the demand for it increases.

**Explain.**

113



In her will the sister of a nurse left all her money to her only sister. After the sister of the nurse died and left some money, the money was distributed according to her wishes. But the nurse did not receive any of this money.

**Give a reasonable explanation.**

114



You were invited to play the following game: every time you have to put half of your money on a bet with a 50% chance of winning and doubling your bet and a 50% chance of losing. Your friend told you that in the long run you would lose all your money. He gave you the following reasoning. Let us say you start with \$100. In the situation 'first win - second lose' (WL) you will end up with \$75. In the situation 'first lose - second win' (LW) you will end up with \$75 again. So in both cases you lose a quarter of your money. Since both situations are equally likely and can be repeated many times, in the long run you will lose all your money.

**Was this reasoning correct?**

115



One evening two couples of old friends were playing a card game. At the end of the evening both couples won the same amount of money.

**How could this be?**

116

\$5 ? \$20

I bet for \$5 that if you give me \$20 I will give you \$50 change.

Would you accept this bet?



117



This puzzle checks your ability to memorise information. Read it just **once**. You can do calculations while reading. Answer as **quickly** as possible.

A traveller has sixteen coins in his wallet. Among them there are 5 Australian, 4 French, 3 American, 2 Italian and 1 Greek.



How many coins are there in the wallet?

118

This puzzle checks your mental arithmetic skills. Read it just **once**. You can do calculations while reading. Answer as **quickly** as possible.

A girl went shopping having 18 coins. In the first shop she gave 3 coins and received 2 coins change. In the next shop she gave 5 coins and received no change. In the next shop she gave 1 coin and received 4 coins change. In the next shop she gave 2 coins and received 2 coins change. In the next shop she gave 3 coins and received 1 coin change. In the next shop she did not buy anything but she found a coin on the floor and picked it up. In the last shop she gave 2 coins and received no change.



How many shops did she visit?



Miscellaneous

119



Two people found a box full of golden treasure: coins, watches, spoons and other things. They want to divide the treasure between them immediately.

Suggest a fair way to divide the treasure.

?

120



Prove that
EDUCATION = ENJOYMENT
given:

- Education is the product of Time and Money
- Time is Money
- Money is the root of Enjoyment.

?

Answers

Solutions

Comments

Answers Only

CHAPTER 1. Buying, Selling, Earning and Paying

1. \$2
2. 75c
3. less than \$50/h (\$48/h)
4. 3 hours
5. \$50
6. John
7. 4 cakes
8. 4 qualified builders
9. 92 chickens, 4 goats and 4 sheep
10. See the 'Solutions and Comments' section
11. The friend lost \$30
12. John (\$1 more than Peter)
13. \$402
14. It is impossible to make such a chain
15. Cut the third link
16. \$30
17. See the 'Solutions and Comments' section
18. 50c
19. \$50
20. See the 'Solutions and Comments' section

Answers Only

CHAPTER 2. Comparing and Dividing Money

21. 10c and 5c
22. Zero chances
23. My sister has \$7 and I have \$5
24. 2kg of \$1 golden coins
25. The first backpacker should get all the money
26. \$1 goes to the first person and \$7 go to the second
27. 27 tourists
28. The situation is impossible
29. a) 10; b) 20
30. See the 'Solutions and Comments' section
31. See the 'Solutions and Comments' section
32. It is uncertain
33. \$150
34. 100 coins

CHAPTER 3. Coins in Wallets, Boxes and on Tables

35. 9 fake coins
36. 5 coins
37. 33 coins
38. 27 coins
39. a) 2; b) 4
40. See the 'Solutions and Comments' section
41. The same
42. See the 'Solutions and Comments' section
43. See the 'Solutions and Comments' section

Answers Only

- 44. 6 coins
- 45. 3 in a row
- 46. Yes. See the 'Solutions and Comments' section
- 47. See the 'Solutions and Comments' section
- 48. See the 'Solutions and Comments' section
- 49. See the 'Solutions and Comments' section
- 50. See the 'Solutions and Comments' section
- 51. 45
- 52. 2 revolutions
- 53. Yes. See the 'Solutions and Comments' section
- 54. 1 coin
- 55. No
- 56. Yes. See the 'Solutions and Comments' section
- 57. See the 'Solutions and Comments' section

**CHAPTER 4.
Operations with Money**

- 58. \$6
- 59. \$8.25
- 60. 20 times
- 61. Step 3
- 62. \$24.50 and \$0.50
- 63. \$5
- 64. \$12 212
- 65. See the 'Solutions and Comments' section
- 66. 190 exchanges
- 67. See the 'Solutions and Comments' section
- 68. 6 ways

Answers Only

**CHAPTER 5.
Share Market**

- 69. 7 days
- 70. 30c
- 71. 10 shares
- 72. Decreased by 13

**CHAPTER 6.
Money and Percentages**

- 73. \$100
- 74. a) the same; b) option two
- 75. a) option two; b) option two
- 76. Cheaper
- 77. 20%
- 78. 10%
- 79. A 30% cut
- 80. \$8
- 81. 100%
- 82. a) 20%; b) 25%
- 83. 32%
- 84. 60%
- 85. The same
- 86. 250%
- 87. 100%
- 88. 400%
- 89. 40%

Answers Only

- 90. 50%
- 91. 25%
- 92. \$50
- 93. Jennifer
- 94. 28%
- 95. 50%

**CHAPTER 7.
Magic Money**

- 96. 15 weeks
- 97. 11 minutes
- 98. 61 coins
- 99. \$35

**CHAPTER 8.
Weighing Coins**

- 100. See the 'Solutions and Comments' section
- 101. See the 'Solutions and Comments' section
- 102. See the 'Solutions and Comments' section
- 103. See the 'Solutions and Comments' section

Answers Only

**CHAPTER 9.
Money and Logic**

- 104. See the 'Solutions and Comments' section
- 105. See the 'Solutions and Comments' section
- 106. The word 'only'
- 107. See the 'Solutions and Comments' section
- 108. It is uncertain

**CHAPTER 10.
Miscellaneous**

- 109. See the 'Solutions and Comments' section
- 110. A fake
- 111. 58 seconds
- 112. See the 'Solutions and Comments' section
- 113. See the 'Solutions and Comments' section
- 114. No
- 115. See the 'Solutions and Comments' section
- 116. See the 'Solutions and Comments' section
- 117. 16 coins
- 118. 7 shops
- 119. See the 'Solutions and Comments' section
- 120. See the 'Solutions and Comments' section

Solutions and Comments

CHAPTER 1. **Buying, Selling, Earning and Paying**

Puzzle 1. \$2.

The first way. There are 2 deals here. From the first deal the profit is: $\$7 - \$6 = \$1$. From the second deal the profit is: $\$9 - \$8 = \$1$. The total profit therefore is: $\$1 + \$1 = \$2$.

Another way to find the total profit is to calculate the total spendings ($\$6 + \$8 = \$14$) and the total takings ($\$7 + \$9 = \$16$). The difference is the total profit: $\$16 - \$14 = \$2$.

It doesn't matter whether it was the same product or not. It is absolutely not important information like the sort of product, the name of the man or the weather on that day. In problem solving it is important to see the difference between essential and irrelevant information. It is one of the distinctive features of a mathematical way of thinking and mathematical modelling.

Both ways to solve this puzzle look easy, however the majority of people, including professional managers, accountants, and

Solutions & Comments

teachers, give incorrect answers. Only about 20% of people on average give the correct answer on their first attempt. This statistic is from my experience giving this puzzle to a large number of people in different countries over the years. You will see some more statistics in the comments to another four puzzles. The intention is to show those readers, who come up with incorrect solutions, that they are not alone and to encourage them keep going.

There are 3 common wrong answers to this puzzle: \$1, \$3 and 'no profit'. The typical mistake of those who answered \$1 is that they considered 3 deals instead of 2. The 'deal' - sold for \$7 and bought for \$8 - gave them a loss of \$1. But selling for \$7 was a part of the first deal and could not be used again.

Another wrong answer is \$3. It was assumed that there was just one deal: initially bought for \$6 and finally sold for \$9, ignoring what happened in between. Technically it was possible to get the right answer calculating the loss of \$1 from the other 'deal' ($\$7 - \$8 = -\$1$) and adding together \$3 and $-\$1$.

Those who answered 'no profit' probably had their own reasoning but couldn't express it clearly. This was possibly a wrong intuition or a result of guessing.

Puzzle 2. 75c.

From the phrase 'they added their money' we know that both girls had money. If Helen had at least 10c, the girls could buy the card because Jenny needed just 10c to buy it. So Helen had less than 10c, that is just 5c. Since Helen was 70c short to buy the card, the price of the card was 75c.

Puzzle 3. Less than \$50 per hour.

The consultant worked more hours for the second company than for the first to earn the same amount because her hourly rate was less: \$40 compared to \$60. So in the total time that she worked for both companies, the proportion of time with the lower rate was greater, therefore her average hourly rate was less than \$50 per hour.

Since the amount she received from each company was not important (it only must be the same) we can choose any easy amount. Let us say she earned \$120 in each company. It took her 2 hours in the first company and 3 hours in the second. So she spent 5 hours to earn \$240. Dividing \$240 by 5 hours we receive that her average hourly rate for the two jobs was \$48 per hour.

In my experience surveying business studies students and lecturers in different countries less than 10% of participants gave the correct answer. The vast majority chose \$50 per hour as the answer.

Solutions & Comments

Puzzle 4. \$50.

The sport shop owner did not lose anything: in the end he received a real \$50 note. The man walked away with the \$15 T-shirt and \$35 change. So the clothing shop lost a \$15 T-shirt and \$35, that is \$50 in value.

Puzzle 5. 3 hours.

The phrase 'buying with the same propensity' is important, as we all know that sometimes it takes just seconds to spend money! Let us adjust first to \$600. Since \$600 is twice as much as \$300, it will take twice as much time for 3 women to spend this amount: $2 \times 3 = 6$ hours. Since 6 women are twice as many as 3 women, it will take them twice as less time, that is $6/2 = 3$ hours. So the answer is 3 hours.

Puzzle 6. John.

It maybe looks like Nick earns more but simple calculations show that John's monthly pay is always \$10 more than Nick's monthly pay. Let us say their starting salary is \$2000 a month. See the diagram:

Months:	0		1		2		3		4
John:		1000	1000	1000	1000	1000	1000	1000	1000
Increments			10	20	30	40	50	60	70
Nick:			2000		2000		2000		2000
Increments				40		80			120

Solutions & Comments

From the diagram we can see that:

the first month's pay	John - \$2010	Nick - \$2000
the second month's pay	John - \$2050	Nick - \$2040
the third month's pay	John - \$2090	Nick - \$2080
the fourth month's pay	John - \$2130	Nick - \$2120

and so on. So John's pays are \$10 more than Nick's every month.

Puzzle 7. 4 cakes.

Let B be the number of bottles of juice and C be the number of cakes. Then the total purchase can be written as:
 $8B + 11C = 100$.

Both quantities B and C can only be whole numbers. Let us try all possible values for C as the most expensive item.
 If $C=1$ then $B=89/8$ which is not a whole number.
 If $C=2$ then $B=78/8$ which is not a whole number.
 If $C=3$ then $B=67/8$ which is not a whole number.
 If $C=4$ then $B=56/8=7$.

If $C=5$ then $B=45/8$ which is not a whole number.
 If $C=6$ then $B=44/8$ which is not a whole number.
 If $C=7$ then $B=23/8$ which is not a whole number.
 If $C=8$ then $B=12/8$ which is not a whole number.
 If $C=9$ then $B=1/8$ which is not a whole number.
 If $C=10$ or more then B is a negative number which is impossible. Therefore the only solution is: 4 cakes and 7 bottles of juice.

Puzzle 8. 4 qualified builders.

Let Q be the number of qualified builders and U the number of unqualified builders. Then the condition for their total salary can be expressed as:

$$140Q + 80U = 960$$

which can be simplified by dividing by 20

$$7Q + 4U = 48.$$

As with the previous puzzle, there are 2 unknown quantities here, both can only be whole numbers.

Let us try all possible values for Q:

If Q=1 then $U=41/4$ which is not a whole number.

If Q=2 then $U=34/4$ which is not a whole number.

If Q=3 then $U=27/4$ which is not a whole number.

If Q=4 then $U=20/4=5$.

If Q=5 then $U=13/4$ which is not a whole number.

If Q=6 then $U=6/4$ which is not a whole number.

If Q=7 or more then U is a negative number which is impossible.

Therefore the only solution is: there were 4 qualified and 5 unqualified builders on that day.

Puzzle 9. 92 chickens, 4 goats and 4 sheep.

Let C be the number of chicken, G the number of goats, and S the number of sheep bought by the farmer. Then the first condition of buying 100 animals can be written as:

$$C + G + S = 100 \quad (1)$$

The second condition of the total price paid can be written as:

$$5C + 35G + 100S = 1000$$

which can be simplified by dividing both sides by 5

$$C + 7G + 20S = 200 \quad (2)$$

Now subtract (1) from (2):

$$6G + 19S = 100 \quad (3)$$

The above condition has 2 unknown quantities G and S, which obviously can only be whole numbers. Let us try all possible values for S as the most expensive animal.

If S=1 then $G = 81/6$ which is not a whole number.

If S=2 then $G=62/6$ which is not a whole number.

If S=3 then $G=43/6$ which is not a whole number.

If S=4 then $G=24/6=4$.

If S=5 then $G=5/6$ which is not a whole number.

If S=5 or more then G is a negative number which is impossible.

So as we can see the only solution to (3) is: S=4 and G=4.

From (1) we can easily find C=92. So the farmer bought 92 chickens, 4 goats and 4 sheep.

Solutions & Comments

Puzzle 10.

Since the takings were a whole number it must be multiple of 5, that is have 0 or 5 at the end (e.g. \$730, \$735).

Puzzle 11. The friend lost \$30.

When the friend redeemed the ticket he paid \$40 to get a coin worth \$50. Thus he paid a total of \$80 for a coin worth \$50.

Puzzle 12. John (\$1 more than Peter).

Let S be John's salary per day, which is the same as the number of days he worked. Then his salary for the whole period is $S \times S = S^2$ dollars. Similarly, Peter's salary for the whole period is $(S-1) \times (S+1) = S^2 - 1$ dollars. So John earned one dollar more than Peter.

Puzzle 13. \$402.

The length of the chain will be $99 \times 4 \text{ mm} + 6 \text{ mm} = 402 \text{ mm}$. The price \$10/cm is the same as \$1/mm. Therefore the price of the chain is \$402.

Puzzle 14.

It is impossible to make a chain from this wire. If the outer diameter of a circular link is 5 mm then the inner diameter is 1 mm ($5 \text{ mm} - 2 \times 2 \text{ mm}$). But it is impossible to put through the 2-mm thick wire.

Solutions & Comments

Puzzle 15. Cut the third link.

The first night the traveller pays this cut link. The second night he pays 2 joined links and receives the cut link as change. The third night he pays 3 joined links and receives the 2 joined links as change. The fourth night he pays the cut link again. The fifth night he pays the 2 joined links and receives the cut link back. The last night he pays the cut link.

Puzzle 16. \$30.

The blacksmith should cut the three links from one of the 3-link pieces. Using these cut links he will be able to join the other four pieces into one 15-link chain. So it requires 3 cuttings and 3 welds, that is $6 \times \$5 = \30 .

Puzzle 17.

The first purchase (5) is a 1-digit number, the second (25) is a 2-digit number, and the third (125) is a 3-digit number. From the prices we can assume that he is buying 1 item the first time, 2 like items the second time, and 3 like items the third time. They could be for example numberplates for the house.

Puzzle 18. 50c.

The total price consists of the price of the cork plus the price of the bottle. The price of the bottle is the price of the cork and \$9. So the difference between the total price of \$10 and \$9 is equal to double the price of the cork. So the cork costs half of \$1 or 50 cents.

Puzzle 19. \$50.

Let N be the number of customers before the reduction and X be the reduction of the price of the ticket. Then before the reduction the takings were $150N$. After the reduction the price of the ticket was $(150 - X)$, the number of customers $3N$, and the takings were $(150 - X)3N$. The takings have doubled, therefore $(150 - X)3N = 300N$. Dividing both sides by $3N$ we obtain $150 - X = 100$. From here $X = 50$. So the price of the ticket has been reduced by \$50.

Puzzle 20.

This is an old puzzle. There are many versions of it. The servant should have subtracted \$2 from \$27 instead of adding it to receive a meaningful amount. The \$27 contains the \$2 he kept and the \$25 paid to the owner, and the \$30 is made up of the \$3 change and the \$27.

CHAPTER 2.

Comparing and Dividing Money

Puzzle 21. 10c and 5c.

If one of the coins is not a 5-cent coin then the other can be. A surprisingly large number of people, some surveys show about 80%, exclude 5c from consideration and therefore think very hard and finally say that it is impossible. Their misinterpretation comes when they replace the phrase "one of them is not a 5-cent coin" with the phrase "none of them is a 5-cent coin" which has a different meaning.

It is very important to think carefully and understand the exact meaning of the sentence. This skill is useful not only in maths but in all areas of life where we need to follow instructions, rules, guidelines, recipes, laws etc.

Puzzle 22. Zero chances.

If two of the three people receive their own currency then the third person automatically receives his/her own currency. So the situation "exactly two" is impossible.

Things sometimes look complicated from one angle but simple from another. Sometimes it is useful to change the 'direction' of your thinking. In this puzzle, instead of trying to list all possible combinations, one can look to what happened with the third note after two of the people received their own currency. From my experience surveying business studies students and lecturers in different countries fewer than 20% of the participants gave the correct answer. The typical incorrect answer was 'one out of three', which was obtained by misusing the classical definition of probability.

Puzzle 23. My sister has \$7 and I have \$5.

It is easy to use the method of guess and check (sometimes referred to as trial and error) and try some numbers, although, of course, it is possible to set up a pair of simultaneous equations and solve them.

Solutions & Comments

Puzzle 24. 2kg of \$1 gold coins.

2 kg of gold is worth more than 1 kg of gold (assuming that a \$1 gold coin is half the weight of a \$2 gold coin).

Puzzle 25. The first backpacker should get all the money.

A common misconception results in paying \$4 to the first and \$2 to the second backpacker since the first backpacker contributed twice as much rice as the second. The underlying reasoning is that the total 600g of rice 'cost' \$6. It is not true. The third person paid \$6 only for his/her part of the dinner, that is for 200g (we assume that they equally shared the dinner - 200g each). The second backpacker supplied his own 200g so the third backpacker bought 200g from the first backpacker. Therefore the first backpacker should get all the money.

Note. Backpackers are usually sharing people who would not let a fellow backpacker starve. It is reasonable that they would share the rice and not charge, knowing that on another day the third backpacker will share with others. So that \$6 was really just a token appreciation.

Solutions & Comments

Puzzle 26. \$1 goes to the first person and \$7 goes to the second.

As with the previous puzzle, a common mistake is to think that 8 pieces 'cost' \$8 and therefore the first person gets \$3 and the second \$5. But the third person paid \$8 only for a third of the fire (assuming that they used the fire equally).

The whole fire 'cost' \$24 and each piece of wood $24/8 = \$3$. The three should contribute an equivalent of \$8 each. The first person gave 3 pieces, that is equivalent to $3 \times \$3 = \9 . So he/she should receive \$1. The second person gave 5 pieces, that is equivalent $5 \times \$3 = \15 . So he/she should receive \$7.

Note. Mountaineers are usually sharing people who would not let a fellow mountaineer suffer. It is reasonable that they would share the fire and not charge, knowing that on another day the third mountaineer will share with others. So that \$8 was really just a token appreciation.

Puzzle 27. 27 tourists.

There are many ways to solve this puzzle. We consider 2 methods.

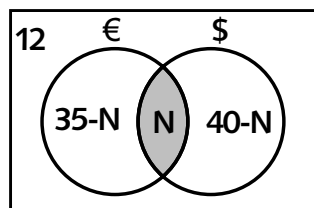
Method 1. We use the following reasoning.

Since 12 tourists had neither € (Euro), nor \$ (American dollars) then $60 - 12 = 48$ tourists had 'only €' or 'only \$' or 'both'. Since 35 of these 48 had €, then $48 - 35 = 13$ did not have €, so they had 'only \$'. Since 40 tourists out of 48 had \$, then $48 - 40 = 8$ did not have \$, so they had 'only €'.

Solutions & Comments

Therefore the remaining $48 - 13 - 8 = 27$ had both € and \$.

Method 2. We use a so-called Venn diagram. Let N be the number of tourists who had both Euro and American dollars. Then $35 - N$ is the number of tourists who had 'only €', and $40 - N$ is the number of tourists who had 'only \$'.
TOTAL = 60.



The total number of tourists was 60, therefore
 $35 - N + 40 - N + N + 12 = 60$.
 From here $N = 27$.

Puzzle 28. The situation is impossible.

Method 1. We use the following reasoning. If we count the number of tourists who had American dollars, British pounds and both currencies we obtain $12 + 16 - 6 = 22$ (by adding 12 and 16 we count the number of tourists who had both currencies twice, that is why we subtract 6). This number is bigger than 20, the total number of tourists, which is impossible.

Method 2. Use a Venn diagram as we did in puzzle 27.

Puzzle 29. a)10; b) 20.

Method 1. We use the following reasoning. Since 30 students had American dollars, 10 students did not have American dollars. Each of these 10 students could either have British pounds or not. Let us consider the extreme cases.

Solutions & Comments

1) If all the 10 students had British pounds, then the remaining 10 students who had British pounds were among those who had American dollars. Therefore the number of students who had both currencies was 10.

2) If none of the 10 students, who did not have American dollars, had British pounds then all 20 students with British pounds were among those with American dollars. Therefore the number of students who had both currencies was 20.

Method 2. Use a Venn diagram as we did in puzzle 27.

Puzzle 30.

One of the fathers was also a son (grandfather/father/son). There were only 3 people.

Puzzle 31.

As with the previous puzzle, this would happen if there were 3 people: son, father, and grandfather. Among them there were 2 sons. One of them did not increase his money since he was also a father: he received \$1000 and gave \$1000. The other son increased his money by \$1000. So the two sons together increased their combined money by only \$1000.

Puzzle 32. It is uncertain!

Puzzle 33. \$150.

It is best to start from the last person and work backwards by adding instead of taking away and doubling instead of halving. Since the fourth person has taken a half of what was left and the **last** \$5 it means that the amount left after that was zero. So the amount after the third person was $\$5 \times 2 = \10 . To calculate the amount left after the second person we need to add \$5 and multiply by 2. So the amount was $(\$10 + \$5) \times 2 = \$30$ after the second person. Similar, the amount after the first person was $(\$30 + \$5) \times 2 = \$70$. So the initial amount was: $(\$70 + \$5) \times 2 = \$150$.

Puzzle 34. 100coins.

Eventually Garry made 50 coins more than he had to make, so Steve made 50 coins less than he had to make. Therefore Garry made 100 coins more than Steve.

CHAPTER 3.**Coins in Wallets, Boxes and on Tables****Puzzle 35. 9 fake coins.**

There can't be two or more real coins because in this case we would be able to select a pair of two real coins which contradicts the second condition: at least one of **any** two coins is fake. So the number of real coins is less than two which can be either 0 or 1. From the first condition (at least one of the coins is real) we conclude that there is one real coin. Therefore the other 9 are fake.

From my experience giving this puzzle to many business studies students and lecturers in different countries over the years, less than 15% of people gave the correct answer. The common wrong answer was 5. Surprisingly many participants just played with the numbers 10 and 2 trying different arithmetic operations to get an answer that made sense to them.

Puzzle 36. 5 coins.

In puzzles like this it is helpful to consider the 'worst' possible scenario: the first coin is from New Zealand, the second is from Australia, the third is from New Zealand, the fourth is from Australia (not necessarily in this order). So far there are 2 New Zealand and 2 Australian coins. No matter what the next coin is the number of coins of the same country will be 3. So the answer is 5 coins.

Puzzle 37. 33 coins.

As with the previous puzzle, consider the 'worst' possible scenario: you select 5 UK coins, 9 New Zealand coins, 9 Australian coins and 9 American coins in any order. Then, no matter which coin is next - American, Australian or New Zealand - you will get 10 coins of the same country. So the answer is: $5 + 9 + 9 + 9 + 1 = 33$.

Puzzle 38. 27 coins.

As with the previous 2 puzzles, consider the 'worst' possible scenario: you select all 15 American and 10 Australian coins in any order. The next two must be New Zealand coins. So the answer is: $15 + 10 + 2 = 27$.

Solutions & Comments

Puzzle 39. a)2; b)4.

Among 10 coins there are 8 \$1 coins and 2 not \$1 coins. Each of the 2 not \$1 coins can be either fake or real. If both not \$1 coins are fake, then 2 \$1 coins are fake. If both not \$1 coins are real, then all 4 fake coins are \$1 coins. So the smallest possible number of fake \$1 coins is 2, and the biggest possible number of fake \$1 coins is 4.

Puzzle 40.

One of the people receives a coin in the wallet.

Puzzle 41. The same.

The number of silver coins in the first, "golden" box is the same as the number of golden coins in the second, "silver" box. When we put one scoop of the mixture back to the "golden" box, some of the golden coins are replaced by the silver coins. It means, that those golden coins, replaced by the silver coins, are in the "silver" box. Actually we can repeat this procedure (take one scoop from one of the boxes, put into the other, mix and then take one scoop of the mixture and put it back into the first box) many times and always the number of silver coins in the first, "golden" box will be the same as the number of golden coins in the second, "silver" box.

Solutions & Comments

Puzzle 42.

From the condition of the puzzle we know that each wallet should contain a non-zero number of coins, that is there is no empty wallet. Let us try different numbers from the smallest possible number of coins, that is from 1:

$$1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 = 45.$$

It works. If we try to increase the number of coins in some of the wallets, then we must decrease the number of coins in some other wallets by the same number. Since we have consecutive numbers (from 1 to 9) it is impossible to avoid repeats. So the above way is the only way.

Puzzle 43.

The two solutions are in the tables below. It is easy to obtain them using the method of guess and check.

	FULL BOXES	HALF-FULL BOXES	EMPTY BOXES
FIRST DAUGHTER	3	1	3
SECOND DAUGHTER	2	3	2
THIRD DAUGHTER	2	3	2

	FULL BOXES	HALF-FULL BOXES	EMPTY BOXES
FIRST DAUGHTER	3	1	3
SECOND DAUGHTER	3	1	3
THIRD DAUGHTER	1	5	1

Solutions & Comments

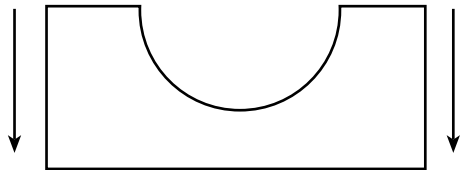
Puzzle 44. 6 coins.

It is possible to provide a rigorous mathematical proof using geometry. But it is much easier and faster to play with real coins to get the answer.

Puzzle 45. 3 coins in a row.

Puzzle 46. Yes.

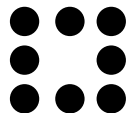
You can do it by folding the paper as it is shown below and pulling the ends down to stretch the semicircle into a straight line.



By doing this the length of the hole becomes a half of the circumference which is approximately 50% more than the diameter. It is enough to put a 20c coin through.

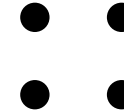
Puzzle 47.

The first way is obvious:



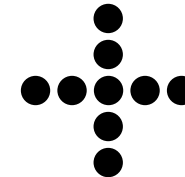
Solutions & Comments

The second way is to put one coin on the top of each of the four coins (a bit of lateral thinking):

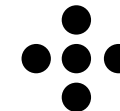


Puzzle 48.

The first way is to put the tenth coin on the top of the coin in the centre:



The second way is to put a coin on the top of each of the five coins:



Puzzle 49.

The first way:



The second way:



Solutions & Comments

Puzzle 50.

The only way this can be done is to form a regular triangular pyramid. You could place 3 coins on a table in the shape of an equilateral triangle and one on the top of a glass sitting upside down in the centre of the triangle formed by the three coins.

Puzzle 51. 45 squares.

Just careful counting. Don't forget about the squares that are positioned obliquely.

Puzzle 52. 2 revolutions.

It is possible to prove this fact using geometry, but the easiest way is to actually practise with any two identical coins.

Puzzle 53. Yes.

Push the cork inside (a bit of lateral thinking).

Puzzle 54. 1 coin.

After you have put one coin in the can, the can is not empty any longer.

Puzzle 55. No.

It is impossible unless the diameter of a small coin is at least 62% (to the nearest whole number) of the diameter of the large coin. Proof of this fact requires some geometry, which is beyond the scope and objectives of this book.

Solutions & Comments

Puzzle 56. Yes.

One of the solutions is below:

H	H		T	T
H		H	T	T
H	T	H		T
H	T	H	T	
H	T		T	H
	T	H	T	H
T		H	T	H
T	T	H		H
T	T		H	H

Puzzle 57.

Tip the second box with coins into the second empty box and put it back. You have only moved one box.

CHAPTER 4. Operations with Money

Puzzle 58. \$6.

Remember the order of arithmetic operations: multiplication is before addition. So $\$2 + \$2 \times 2 = \$6$.

Puzzle 59. \$8.25.

Operation 'twice' cancels operation 'half'.

Puzzle 60. 20 times.

Just careful counting.

Puzzle 61. Step 3.

When we are doing calculations with numbers and units we must do the same calculations for both numbers and units. Therefore after squaring the answer is $\$^2 4 = 40,000c^2$. These units ($\2 and c^2) don't have real meaning and of course from there we can't get the equality in step 4.

Puzzle 62. \$24.50 and \$0.50.

Method 1. We use the guess and check method. We can try first \$24 and \$1. The first part is 24 times more than the second. Now let us try to halve the smaller part, that is the

first part is \$24.50 and the second is half a dollar. Dividing a number by a half is the same as multiplying by 2, therefore dividing the first part (\$24.50) by the second (half a dollar) we will get the result 49.

Method 2. Set up a pair of simultaneous equations and solve them.

Puzzle 63. \$5.

By giving \$5 to Jane Chris has \$5 less than before and Jane has \$5 more than before. Therefore the difference between their money is \$10.

Puzzle 64. \$13,212.

12 hundred is 1 thousand plus 2 hundred, and the 1 thousand is added to the 12 thousand.

Puzzle 65.

Normally people select an easy special number among the given numbers. Often it is the smallest or the biggest number. Since the natural numbers don't have a biggest number it is sensible to choose the smallest natural number, that is 1. If both have the same reasoning they both would choose \$1 and receive the prizes. Of course there could be other 'special' amounts such as \$100, \$1000, or \$1,000,000.

Puzzle 66. 6 ways.

It is easy here to use systematically the method of guess and check. All possible ways are listed in the table below:

Ways	\$5	\$10
first way	10	0
second way	8	1
third way	6	2
fourth way	4	3
fifth way	2	4
sixth way	0	5

Puzzle 67. 190 exchanges.

Each of the 20 businessmen gave a coin to all others (apart from himself), that is to 19 people. So the total number of 'givings' was $20 \times 19 = 380$. Since one exchange is two 'givings' the total number of exchanges was $380 \times 2 = 190$.

Puzzle 68.

To be able to reach \$100 you must ensure you reach \$89. In this case the difference is \$11 and whatever amount your partner adds you are able to reach \$100 by adding the amount less than \$10. To reach \$89 you must reach \$78. Always keep the difference of \$11. This gives the following sequence: 1, 12, 23, 34, 45, 56, 67, 78, 89, 100. Therefore to win you should begin with \$1 and add money to follow the above sequence.

CHAPTER 5. Share Market**Puzzle 69. 7 days.**

The price was increasing by 5c a day. After six days the price was \$3.50. The next morning the price was increased by 20c and reached \$3.70.

Puzzle 70. 30c.

The difference between the prices reduced by 15c per day. One day before the prices became the same the difference was 15c and two days before it was 30c.

Puzzle 71. 10 shares.

If I have S shares, than my brother has $3S$ shares, our father has $9S$ shares and our grandfather has $27S$ shares. All together we have $S + 3S + 9S + 27S = 40S$ shares. The total number of shares is 400, therefore $S = 10$. So I have 10 shares.

Puzzle 72. Decreased by 13.

Method 1. We use the following reasoning. The total number of transactions is the sum of the number of successes and number of failures. Since the number of successes increased by 3, then decreasing the number of failures also by 3 will give us the same total number as yesterday. To have the total

Solutions & Comments

number 10 less than yesterday we need to decrease further the number of failures by 10 (we cannot change the number of successes). So the total decrease of failures is $3 + 10 = 13$.

Method 2. Set up a pair of simultaneous equations and solve them.

CHAPTER 6. Money and Percentages

Puzzle 73. \$100.

The initial price is 100%. If the price was reduced by, let us say \$20, and it was 20%, then multiplying both numbers by 5 we obtain 100% is \$100.

Puzzle 74. a) The same.

If C is the price of the car then the first option means that after a 15% reduction the car costs 85% of the initial price and after charging a 10% commission the car costs 110% of the price after the reduction.

That is for the first option we have:

$$C \times \frac{85}{100} \times \frac{110}{100}$$

Similar for the second option we have:

$$C \times \frac{110}{100} \times \frac{85}{100}$$

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We can see that the numbers in the above two products are the same, so the results are the same (we know that $2 \times 3 = 3 \times 2$).

From my experience giving this puzzle to business studies students and lecturers in different countries over the years students usually perform better than their lecturers. One of the reasons could be that the majority of the students still remember school mathematics or have not lost an intuitive feeling for the commutative property of multiplication ($a \times b = b \times a$).

b) option 2.

A 10% commission is taken from the price of the car before the reduction, that is from a bigger amount than that after the reduction.

Puzzle 75. a) Option 2.

Just simple calculations: if C is the initial price of the computer then the first option gives you

$$C \times \frac{90}{100} \times \frac{110}{100} = C \times \frac{99}{100}, \text{ that is 99\% of the initial price.}$$

The second option gives you

$$C \times \frac{80}{100} \times \frac{120}{100} = C \times \frac{96}{100}, \text{ that is 96\% of the initial price.}$$

b) option 2.

The first option gives the salesman 10% of the 90%, that is 9% of the initial price. The second option gives him 20% of the 80%, that is 16% of the initial price.

Puzzle 76. Cheaper.

10% of the increased price is bigger than 10% of the initial price. So we subtract a bigger number from the increased price than the difference between the increased and initial prices. For example, if the initial price was \$100, it went to \$110, then it fell by 10% of \$110 or \$11 to \$99.

Puzzle 77. 20%.

After a 25% increase the new price is $100\% + 25\% = 125\%$ of the initial price, that is $P \times \frac{125}{100}$ or $P \times \frac{5}{4}$.

To get the initial price we must multiply the new price by $\frac{4}{5}$, that is $P \times \frac{5}{4} \times \frac{4}{5} = P$.

Multiplying by $\frac{4}{5}$ is the same as multiplying by $\frac{80}{100}$, that is taking 80%. It means that reduction is $100\% - 80\% = 20\%$.

Puzzle 78. 10%.

From the solution to the previous puzzle we can see that Helen has 20% less money than John (John's money is 100%). To equalise their money John should give Helen half of the difference, that is 10% of his money.

Puzzle 79. A 30% cut.

Just simple calculations: a 30% cut means that the new price is 70% of the initial price. A 20% cut followed by a 10% cut gives us: $P \times \frac{80}{100} \times \frac{90}{100} = P \times \frac{72}{100}$ or 72% of the initial price.

Puzzle 80. \$8.

\$6 is 75% or three quarters of the price of the book. One quarter is \$2 and the book's price is \$8.

Puzzle 81. 100%.

We compare \$50 with \$25 therefore \$25 is 100%. \$50 is greater than \$25 by \$25 or 100%.

Puzzle 82. a) 20%; b) 25%.

In both cases the difference is \$10. By 100% we always mean the quantity we compare with. In the first case it is \$50 therefore \$10 is one fifth of \$50 or one fifth of 100%, that is 20%. In the second case \$40 is 100% and \$10 is one quarter of \$40 or one quarter of 100%, that is 25%.

Solutions & Comments

Puzzle 83. 32%.

Just simple calculations. After the first increase the salary (S) was $s \times \frac{110}{100}$. After the second increase the salary became $s \times \frac{110}{100} \times \frac{120}{100}$ or $s \times \frac{132}{100}$, that is 132%.

So the salary increase compared to the starting salary was $132\% - 100\% = 32\%$.

Puzzle 84. 60%.

One way to solve this puzzle is to select any two numbers such that the second is 150% bigger than the first. This means that the second is 250% of the first, or two and a half times bigger: e.g. 40 and 100. It is easy to see that 40 is 60% less than 100.

In general: the condition that the second book (S) is dearer than the first (F) by 150% means that we compare with the first book and therefore the first book is 100% and the second is $100\% + 150\% = 250\%$ of the price of the first book, that is $S = F \times \frac{250}{100}$ or $S = F \times \frac{5}{2}$.

To get F from here we must multiply both sides by $\frac{2}{5}$, therefore $F = S \times \frac{2}{5}$.

Multiplying by $\frac{2}{5}$ is the same as multiplying by $\frac{40}{100}$ or taking 40%. So the first book is 40% of the price of the second and therefore 60% cheaper than the second.

Solutions & Comments

Puzzle 85. The same.

One of the easiest ways to solve this puzzle is to choose any simple number, e.g. \$100. After increasing the price by 100% (\$100 is 100%) the new price is \$200. By decreasing the new price by 50% (\$200 is 100%) we get \$100, that is the initial price. It is not hard to show this for any numbers as we did in the previous two puzzles.

Puzzle 86. 250%.

As with the two previous puzzles, one way to solve this puzzle is to choose simple numbers, e.g. John's hourly wage is \$40 per hour and Peter's hourly wage is \$100 per hour. Obviously John's wage is 40% of Peter's wage because we compare with Peter's wage and his \$100 wage is 100%. When we compare with John's wage his \$40 wage is 100%. Peter's wage is two and a half times higher, that is 250%. It is not hard to show this for any numbers as we did in puzzles 83 and 84.

Puzzle 87. 100%.

As previously choose simple numbers, e.g. Andrew has \$100 and Murray has \$200. Since we compare with Andrew's amount, his money (\$100) is 100%. So Murray has \$100 more or 100% more. It is not hard to show this for any numbers as we did in puzzles 83 and 84.

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Puzzle 88. 400%.

As previously choose simple numbers, e.g. Bill has \$100 and Jack has \$500. Since we compare with Bill's amount, his money (\$100) is 100%. So Jack has \$400 more or 400% more. It is not hard to show this for any numbers as we did in puzzles 83 and 84.

Puzzle 89. 40%.

If we had \$5 and \$7, then the increase would be \$2 and the percentage increase would be 40% ($\frac{\$2}{\$5} \times 100\% = 40\%$). With 5% and 7% it can be confusing since the unit for the interest rate is %. In our case the absolute increase is $7\% - 5\% = 2\%$ and the percentage increase is 40%. To avoid misunderstanding when we are talking about the absolute increase in such phrases as "the interest rate has increased by 2%" it is agreed in some countries to say "the interest rate has increased by 2% points".

Puzzle 90. 50%.

For many people this is a surprising answer. But let us calculate. Since John spends all his salary (S) on goods and services (G) then we can conclude that $S = G$. After his 20% pay increase his salary became 120% of S or $\frac{120}{100} \times S$ or $\frac{120}{100} \times G$ since $S = G$. At the same time the costs of all goods and services decreased by 20%, so John's spending became 80% of G or $\frac{80}{100} \times G$.

Solutions & Comments

Therefore the difference between his new salary and his new spending is $\frac{120}{100} \times G$ minus $\frac{80}{100} \times G$, that is $\frac{40}{100} \times G$, which is exactly half of the cost of his new spending ($\frac{80}{100} \times G$), that is 50%.

One can get the same answer more easily by selecting any simple number for S and G, say, \$100.

Puzzle 91. 25%.

The simplest way to solve this puzzle is to choose an easy number, say, \$100 as the marked price. Then with a 20% discount the man paid \$80 for the product. Selling the product at the marked price the man received \$20 profit, which is a quarter of what he paid (\$80), that is 25%.

Of course, it is possible to use P as a marked price and show the result for any number as we did in the previous puzzle.

Puzzle 92. \$50.

Let us calculate how much the mushrooms would weigh a week later. Out of the total weight of 100kg the moisture content is 99% or 99kg and the solid content is 1% or 1kg. When drying the moisture is vaporising but the solid content remains the same, that is 1kg. In a week's time the moisture content would be 98% and the solid content 2%. So 2% is 1kg and the total weight (100%) is 50 times more: $2\% \times 50 = 100\%$ or $1\text{kg} \times 50 = 50\text{kg}$. Therefore if selling for the same price of \$1 per kg the farmer would receive \$50 instead of \$100. So he would lose $\$100 - \$50 = \$50$.

Solutions & Comments

In general: vaporising from 99% of moisture content to 98% of moisture content will result to 50% of loss of **any** initial weight (and 50% loss of money if selling for the same price). Let W be the initial weight. Then the initial solid content is 1% or $\frac{W}{100}$.

After drying out to 98% of moisture content the solid content has the same weight of $\frac{W}{100}$ and makes 2% of the new total weight. The new total weight (100%) is 50 times more than 2%, therefore the new total weight is $\frac{W}{100} \times 50 = \frac{W}{2}$.

So the total weight is half or 50% of the initial weight.

Puzzle 93. Jennifer.

Let A be a number of documents Jennifer makes per day and B be an amount paid for each document made. Then Jennifer's salary per day is $A \times B$.

Lucy receives 10% less than Jennifer, that is 90% of B or $\frac{90}{100} \times B$, but she makes 10% more than Jennifer, that is 110% of A or $\frac{110}{100} \times A$.

Therefore Lucy's salary per day is $\frac{110}{100} \times A \times \frac{90}{100} \times B$ or $\frac{99}{100} \times A \times B$ or 99% of $A \times B$.

So Lucy earns 1% less than Jennifer.

Solutions & Comments

Puzzle 94. 28%.

When Mary got a 10% discount from the first voucher, she paid 90% of the normal price P , that is $P \times \frac{90}{100}$.

The second discount of 20% means that she paid 80% of the price after the first discount, that is $P \times \frac{90}{100} \times \frac{80}{100}$ or $P \times \frac{72}{100}$ or 72% of the normal price.

Therefore her total discount was $100\% - 72\% = 28\%$.

Puzzle 95. 50%.

Let MP be the marked price (before the reduction).

Let SP be the selling price, which after a 20% reduction was 80% of MP or $SP = MP \times \frac{80}{100}$.

Let CP be the cost price. Then 20% profit means that after selling the dress the owner received 120% of CP or $SP = CP \times \frac{120}{100}$.

Obviously $MP \times \frac{80}{100} = CP \times \frac{120}{100}$.

Cancelling 100 and dividing both sides by 80 we obtain $MP = CP \times \frac{120}{80}$ or $MP = CP \times \frac{3}{2}$ or $MP = CP \times \frac{150}{100}$.

So the marked price MP (before the reduction) was 150% of the cost price CP , therefore the intended profit was $150\% - 100\% = 50\%$.

One can get the same answer more easily by selecting any simple number for the cost price, say, \$100.

CHAPTER 7. Magic Money

Puzzle 96. 15 weeks.

To get one million dollars in week 16 the businessman had to double half a million dollars. So a week before, that is in week 15, he had half a million dollars.

Puzzle 97. 11 minutes.

Since we start with 2 coins, we have one step less, namely converting the first coin into two, which takes one minute. So we save one minute from the total time in the case when we start from one coin (12 minutes). So the answer is $12 - 1 = 11$ (minutes).

Puzzle 98. \$35.

Let us start from the end of the story. From the fact that the man gave the last \$40 to the magician after crossing the bridge for the third time we know that he had half of that money, that is \$20, before crossing the bridge for the third time. That \$20 was left after he paid \$40 to the magician once he crossed the bridge for the second time. So before he paid \$40 to the magician he had $\$20 + \$40 = \$60$ which he received by doubling \$30. So he had \$30 before crossing the bridge for the second time. That \$30 was left after giving the \$40 'fee' to the magician after crossing the bridge for the first time. So he had $\$30 + \$40 = \$70$ after crossing the bridge for the first time, which he received by doubling \$35, his initial amount.

Puzzle 99. 61 coins.

We start counting time from the first coin fallen from the sky. The time starts from zero. So at the moment of time 0 seconds we get the first coin, at the moment of time 1 second we get the second coin, and so on. At the moment of time 60 seconds we will get the 61st coin.

CHAPTER 8. Weighing Coins

Puzzle 100.

Puzzle 100. We divide the nine coins into 3 equal parts. Then we put 2 of these parts on the balance scale - 3 coins on each side of the scale. If the scale is in equilibrium then the fake coin is among the three coins that we did not put on the scale. If the fake coin is on the scale then it is among the three coins that have lighter weight. Now we take the 3 coins that contain the fake coin, put one aside and one on each side of the scale. If the scale is in equilibrium then the fake coin is the coin that we put aside. If the scale is not in equilibrium then the lighter coin is the fake.

Similarly one can show that it takes at most 2 weighings to identify the fake coin if the total number of coins is 4, 5,...8.

Puzzle 101.

We divide the 27 coins into 3 equal parts - 9 coins in each. Then we put 2 of these parts on the balance scale - 9 coins on each side of the scale. If the scale is in equilibrium then the fake coin is among the nine coins that we did not put on the scale. If the fake coin is on the scale then it is among the nine coins that have lighter weight. So after the first weighting we identified the nine coins that contain the fake coin. Then we do another two weightings as we did in the previous puzzle to identify the fake coin.

Similarly one can show that it takes at most 3 weighings to identify the fake coin if the total number of coins is 10, 11, ..., 26.

Puzzle 102.

Dividing the 81 coins into three equal parts and using the approach similar to the previous two puzzles we can identify the 27 coins that contain the fake coin. So we have one additional weighing to the previous puzzle.

Similarly one can show that it takes at most 4 weighings to identify the fake coin if the total number of coins is 28, 29, ..., 80.

Puzzle 103.

You need to select 1 coin from the first wallet, 2 coins from the second wallet, 3 coins from the third wallet, and so on ..., 10 coins from the 10th wallet. The total number is 55 coins. If all coins were real then the total weight would be $55 \times 10\text{g} = 550\text{g}$. Since some of the 55 coins are fake and 1g heavier than a real coin, then the total weight will be more than 550g. The difference between the total weight and 550g is the same as the number of the wallet. For example, if wallet 6 contains fake coins then the total weight will be 556g: we have an extra 6 grams compared to 550g as we put 6 fake coins from wallet 6 each weighting 1g more than a real coin.

There is an interesting, though unverified, story about this puzzle. During the Second World War a group of English scientists from a military research centre spent lots of time solving this puzzle. The director of the centre considered this as time wasting and therefore as an activity undermining the military power of England. Some of the scientists suggested that they throw leaflets with this puzzle on it from a plane above German territory to cause the same damage in Germany.

CHAPTER 9. Money and Logic

Puzzle 104.

a) The statement is broken if John receives a bonus at work but doesn't buy Telecom shares.

b) Nothing. The statement is a one-way implication. Receiving a bonus at work is a sufficient condition for buying Telecom shares. There could be other sufficient conditions for buying Telecom shares that have nothing to do with his bonus (for example, winning a lottery or at the casino). So he doesn't need to wait for a bonus at work to buy Telecom shares. The statement doesn't prevent John from buying Telecom shares if he doesn't receive a bonus at work.

c) Nothing. The same comments as in b).

In logical puzzles it is very important to avoid false assumptions and overgeneralisations. Don't assume or imagine anything unless you have a clear indication or reason to do so.

Puzzle 105.

a) The statement is broken if John doesn't receive a bonus at work but still buys Telecom shares or if John does receive a bonus at work but doesn't buy Telecom shares. The statement is a two-way implication. The condition for buying Telecom

shares is necessary and sufficient. It means that the **only** situation when John buys Telecom shares is that when he receives a bonus at work. In other words to buy Telecom shares he must receive a bonus and if he receives a bonus he must buy Telecom shares.

b) The fact that John buys Telecom shares means that he received a bonus at work. See the comments in a).

c) The fact that John doesn't receive a bonus at work means that he doesn't buy Telecom shares. See the comments in a).

Puzzle 106. The word 'only'.

The phrase 'he doesn't receive a bonus at work' is the opposite to the phrase 'he receives a bonus at work' and therefore the only alternative to it. It excludes all other options. Of course the statement could be rephrased. For example, John buys Telecom shares regardless of receiving a bonus at work

Puzzle 107.

The situation is really unclear for the leader. He should ask for a special instruction for himself.

Puzzle 108. It is uncertain.

It is impossible to answer the question since whisky was not mentioned as a favourite drink of one of the businessmen. The fact that you were given just 30 seconds to solve this puzzle was a hint that the puzzle is not a usual logical puzzle of this sort.

CHAPTER 10. Miscellaneous

Puzzle 109.

Because there are 5 more dollars in 1995 dollars than in 1990 dollars. Some people think that 1995 and 1990 are years when the dollar notes or coins were produced.

Puzzle 110. A fake.

The abbreviation BC (Before Christ) was introduced after Jesus Christ death. So in the year 369 BC people could not use this system and put 369 BC on the coin.

Puzzle 111. 58 seconds.

The 29th cut will cut the 29th 1-cm piece of the wire leaving the remaining 30th 1-cm piece.

Puzzle 112.

When the price for a cheap but essential food (e.g. bread, rice, potato) is increasing people still buy it because it is essential. It means they spend more money on it. So they have less money to buy more expensive food (e.g. meat, fish). They must substitute the expensive food with the cheap food, (e.g. bread, rice, potato), which was increased in price. That is why the demand for it increases.

Puzzle 113.

The nurse was a man (a bit of lateral thinking).

Puzzle 114. No.

Apart from WL and LW there are two other cases when you play this game twice: WW and LL. In case WW you will end up with \$225 and in case LL you will end up with \$25. All the four cases are equally likely. And although you lose the money in three of them and win only in one, the total amount you win in all cases is zero ($\$125 - \$25 - \$25 - \$75 = 0$). It is not hard to show that this will be true for any initial amount of money and for any number of times you play the game.

Puzzle 115.

The two couples were playing against other couples, not against each other (a bit of lateral thinking).

Puzzle 116.

You should not accept the proposed bet because I would be happy to lose the bet, give you \$5 and keep your \$20.

Puzzle 117. 16 coins.

See the first sentence. If you add up the listed coins you will get 15 coins. But those coins were just 'among them' and of course did not exclude any other coin or coins.

Solutions & Comments

Puzzle 118. 7 shops.

Most people make calculations of the amount of money. Since the condition was 'read it just once' it is not easy for some people to recall the number of shops visited. It is a good joke at a party when people just listen to the puzzle, and can't read it.

Puzzle 119.

One fair way could be: one person divides the treasure into two parts and the other chooses one of the parts. According to The Bible, King Solomon used this way to sort out a similar situation.

Puzzle 120.

The 'proof' is below.

The first statement 'Education is the product of Time and Money' can be written as the equation:

$$\text{EDUCATION} = \text{TIME} \times \text{MONEY}$$

The second statement 'Time is Money' is the equality:

$$\text{TIME} = \text{MONEY}$$

Substitute MONEY instead of TIME into the first equation:

$$\text{EDUCATION} = (\text{MONEY})^2$$

Solutions & Comments

The third statement 'Money is the root of Enjoyment' can be written as:

$$\text{MONEY} = \sqrt{\text{ENJOYMENT}}$$

Substituting into the previous equation we obtain:

$$\text{EDUCATION} = (\sqrt{\text{ENJOYMENT}})^2$$

But squaring cancels the square root, therefore

$$\text{EDUCATION} = \text{ENJOYMENT}.$$

About the Author

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