Designed to be an effective "brain gym" as well as to educate and entertain, *Shape Puzzles* delves into the wonderful realm of shapes in the natural world around us and within us. This book will appeal to all readers from 12 years and above and will benefit those who like to be challenged. While the puzzles are qualitative in nature, emphasis is placed on different ways of thinking, paradoxical reasoning and common sense. Most of the puzzles require no calculation or very simple calculations and appear easy until you try to solve them. Answers and solutions are given at the end of the book.

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John Buckeridge Sergei Gulyaev
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In the same series:
Number Puzzles
Science Puzzles

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CONTENTS

PREFACE

We are surrounded by variety in shapes. Shapes may be physical objects in the real world or they may exist in the abstract world of mathematics. We discover them, invent them, investigate them and may ultimately use them. Our observations have led us to expect some consistency in shapes, and this in turn has led us to classify many objects on the basis of their geometry. But why are certain things remarkably consistent in form? And why are they “that particular shape”? There are times when our intuition fails and we are challenged by their mystery.

This book delves into the wonderful realm of shapes. We challenge you to evaluate shapes in the natural world around us, within us and in the heavens above us. In addition, we provide some more abstract problems for you to solve, some of which have surprising and complex answers. As with our previous two books *Science Puzzles* and *Number Puzzles* published by Times Editions—Marshall Cavendish, we don’t assume any special knowledge from the reader. This book is for anyone who is interested in science and maths, with wide use anticipated in secondary schools. Like our earlier titles, the book may be used to stimulate discussion and debate, both at school and at home, or just to have fun. It is designed to be an effective ‘brain gym’, one that benefits all who are challenged to think about its contents.

“The puzzles in the book are designed to both educate and entertain.”

Most of the puzzles have qualitative character and require no or very simple calculations. The emphasis is on different ways of thinking (logical, lateral, spatial, critical, holistic and versatile), paradoxical reasoning and common sense. Most of the puzzles appear simple until you try to solve them. Some of them are easy but the majority are quite tricky, thought provoking, challenging and even teasing. As in *Science Puzzles* and *Number Puzzles*, we have deliberately mixed all the puzzles.

The puzzles in this book are from a variety of different sources. Some were picked up from conversations with colleagues and friends, some were created or converted into a different context by the authors and some were just questions often asked by children. In most cases, it is impossible to identify

the author of any particular puzzle as one can encounter many different versions of it in different books published in many languages. Often the authors of famous old puzzles are unknown.

Our goal is to illustrate and popularise different styles of thinking using the shape context and to open the door to the wonderful world of shapes. We trust you will enjoy it—we sure had fun writing it!

John Buckeridge
Sergei Gulyaev
Sergiy Klymchuk

August 2005

THE PUZZLES

A Crystal Shape

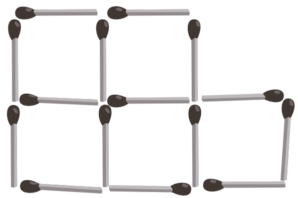
Common table salt (sodium chloride) crystals may be grown in the kitchen by dissolving as much salt as you can in a small pot of boiling water. When no more salt dissolves, remove the pot from the heat source, pour away the liquid and let the pot and its contents cool down. Once cooled, you will see many white salt crystals in your container. Use a magnifying glass to look at them.

What shape are the crystals? Are they all the same shape?

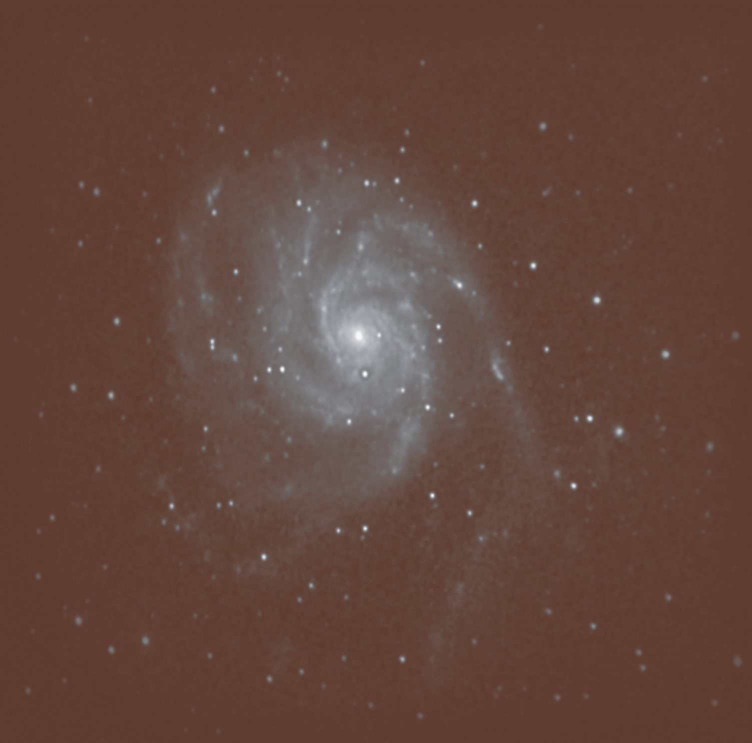
1

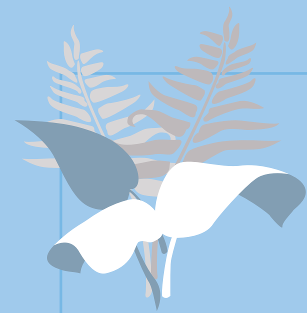
Reducing the Number of Squares

Using 15 matches, place them in the pattern shown below. Now remove only 3 matches to leave 3 out of 5 squares.



2





Leaves
Why are plant leaves relatively flat?

3

Changing the Number
Add just one symbol to the number 9 written in Roman numerals —IX—to make the number 6.

VIII IX X XI

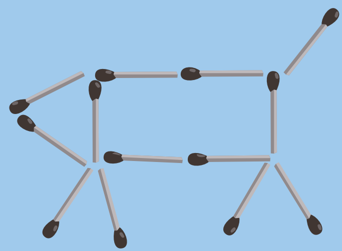
4

Spiral Shape of Galaxies
A spiral galaxy consists of stars rotating around the galaxy's centre, forming a "stellar disc" comprising billions of stars. A typical spiral galaxy can be seen in the picture opposite. Why does the stellar disc of the galaxy have a spiral shape? What is between the spiral arms?

5

Changing the Direction

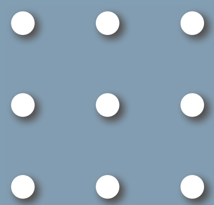
Move just 2 matches to make the dog face the opposite direction with its tail still pointing up.



6

Crossing the Dots

Cross all 9 dots using 4 straight lines without lifting your pen from the paper.



7

Changing the Shape

Cut a square (e.g. as below) into pieces to form a shape known as the Greek cross (shown in the bottom sketch). You must use all parts of the square you have cut! First try to do it with 5 pieces, then with 4 pieces.

8

Eratosthenes' Problem

The ancient philosopher Eratosthenes of Cyrene (276–194 B.C.) lived in Alexandria and worked as a librarian in the famous Alexandria Library. To work out the size of the Earth, Eratosthenes used two well-known facts:

1. On Midsummer's Day, the Sun's rays reach the bottom of the deepest well in Syene (now Aswan on the River Nile in Egypt).

2. Syene is 5000 stadia (an ancient Greek measurement of length), which is 800 kilometres (using today's metric system), south of Alexandria.

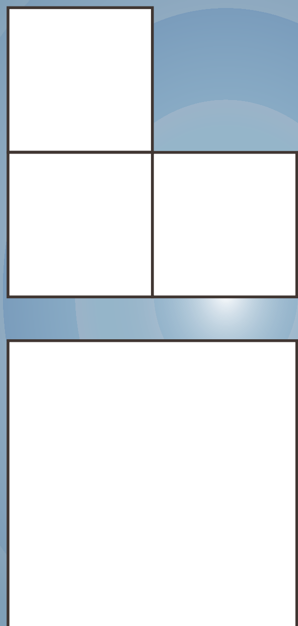
Eratosthenes also used his own measurements of the height of the Sun at noon on Midsummer's Day, which he performed on the roof of Alexandria Library. The Sun happened to be $\frac{1}{50}$ part of the circle, or 7.2° away from the zenith (the point in the sky directly above his head).

Living 2200 years after Eratosthenes, can you solve the problem of the size of the Earth solely on the basis of this data?

9

Dividing the Figures

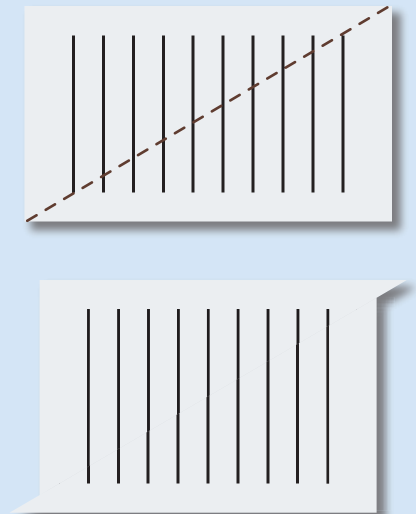
Using straight lines, divide the first figure into 4 identical parts and the second into 5 identical parts.



10

A Missing Line

Draw 10 equal vertical lines on a rectangular piece of paper. Cut along the dashed diagonal line. Then move the lower part of the rectangle to the left by the distance between 2 lines. You will see that one line has disappeared; you can see only 9 lines. How can you explain this?



11

A Missing Square

Photocopy this page and cut out the figures below along the solid lines in Diagram 1 and reassemble them as shown in Diagram 2. You will be amazed to see an extra empty space has appeared, which is shown by the shaded square. This means that the area of the figure in the second diagram is 65 - 1 = 64 square units whereas the area of the figure in the first diagram is 65 square units. So the area of one square unit has vanished. How can you explain this?

If you would rather not cut out your own triangles, simply compare the two diagrams below.

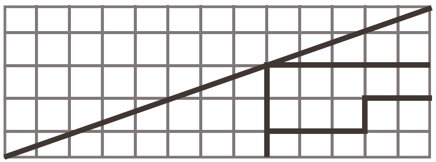


Diagram 1

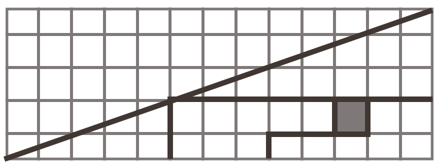


Diagram 2

12

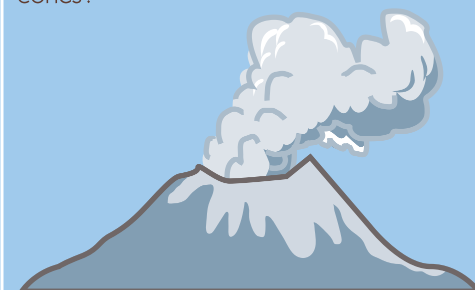
Snowflakes

Why do snowflakes form such wonderful shapes?

13

Volcanic Shapes

Why are some volcanoes, like Hawaii's Kilauea, rather flat, shield-like structures while others, such as Mt. St. Helen's in the United States, are characterised by steep-sided cones?



14

Dividing the Moon
Divide the crescent below into 6 pieces using just 2 straight lines.

15

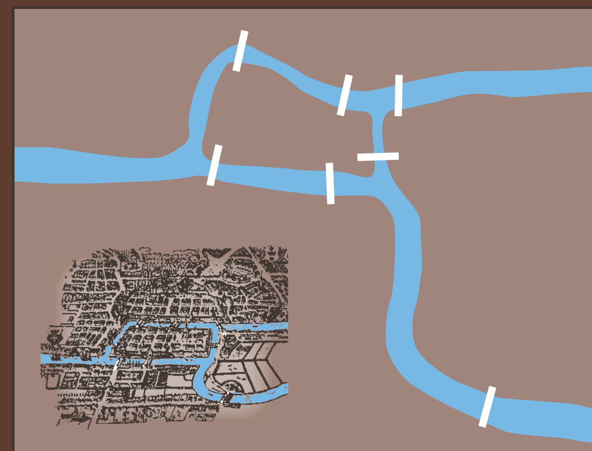
Filling a Barrel
How can you fill a barrel half full with water without using any measurements?

16

Euler's Problem
Leonard Euler (1707–1783), one of the greatest mathematicians, visited Königsberg, Germany. At that time there were 7 bridges in Königsberg that connected the banks of the local river and two islands.

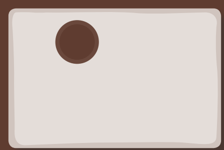
Euler posed the following question: Is it possible to walk across all 7 bridges (shown in red in the lithograph), walking across each bridge just once?

17



Cutting a Cake

A circular chocolate has been placed on a rectangular cake but not in the centre. How can you divide the cake and the chocolate into two equal parts with only one cut?



18

The Nature of Soil

Why is clayey soil generally impermeable, but sandy soil porous?

19

Tubular Worms

Tapeworms and earthworms are both members of the Annelida[1]. Although they have many things in common, earthworms have cylindrical cross sections while tapeworms are flat. Does this difference in shape have any significance?

[1] Annelids are a primitive group of multicellular, segmented organisms that lack any form of mineralised skeleton.

20

Winning Strategy at the Pizzeria

You and a friend are playing the following game at a pizzeria. On a large circular pizza each of you in turn places a slice of salami anywhere on the pizza until the whole pizza is completely covered by slices of salami one layer thick. All of the slices are the same size. The winner is the one who places the last slice. If you start the game, how should you play to win?

21

Animal Size

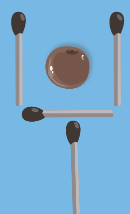
The largest animals, either alive today or known as fossils, lived in water. Why?

22

Dividing Yin and Yang
Divide yin and yang in the ancient oriental symbol below into 4 equal parts of the same shape using only one cut.

23

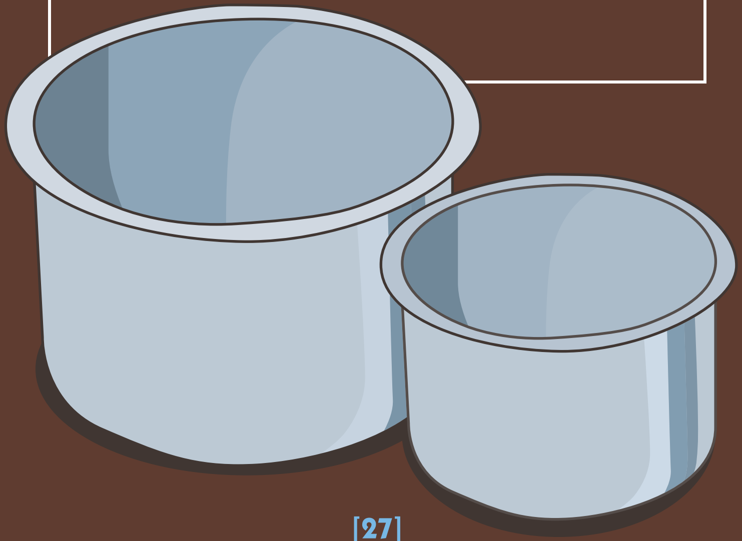
A Berry in a Wine Glass
Move only 2 matchsticks in such a way that the berry will be outside of the wine glass. You can change the position of the glass but not its shape.



24

Two Containers
There are 2 containers of the same shape and thickness and made of the same material. The volume of one of the containers is 8 times greater than the volume of the other. How many times is it heavier?

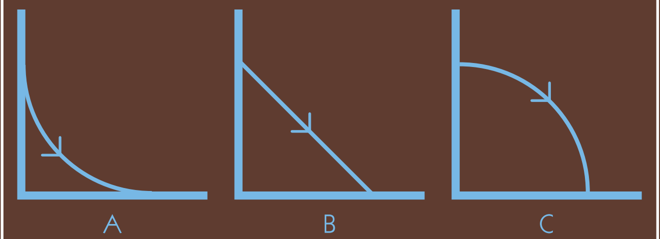
25



A Cat on a Ladder

Imagine a cat sitting half way up a ladder that is placed almost flush with a wall. If we push the base of the ladder fully up against the wall, both the ladder and the cat will most likely fall away from the wall (i.e. the top of the ladder will fall away from the wall).

Part 1: If, by some small chance, the cat manages to stay on the ladder, what will the trajectory of the cat be? A, B or C?

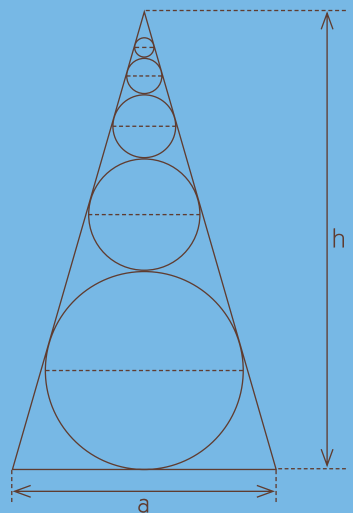


Part 2: Which of the above options represents the cat's trajectory if, instead of the top of the ladder falling outwards, the base is pulled away (with the top of the ladder retaining contact with the wall)? A, B or C?

26

Circles

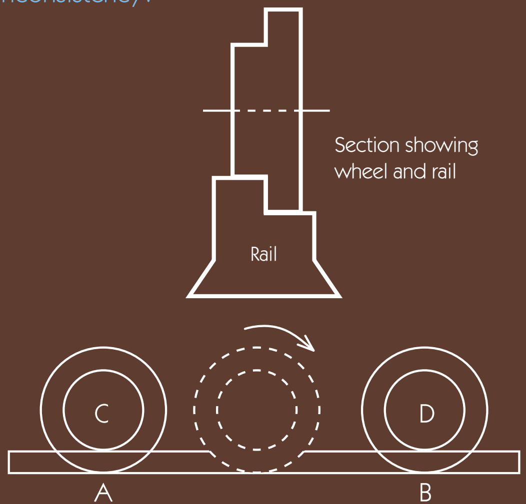
What is the sum of the diameters of the infinitely many circles drawn inside this triangle such that they do not overlap?



27

Wheels

Two wheels are attached to each other and put on the same axis. Both wheels are on the rail. After one rotation, the big wheel covers the distance AB, which is equal to the length of its circumference, and the small wheel covers the distance CD. It is clear that AB = CD but the circumferences are different because the radii are different. Why is there this inconsistency?



28

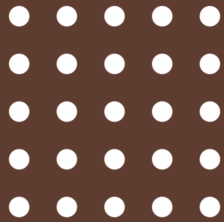
Counting Triangles

How many triangles are there in the figure below?

29

Counting Squares

By joining the dots below, see how many squares you can get.



30

Constructing Triangles

Construct 4 equilateral triangles using only 6 matches.

31

An Extra Square

Photocopy the page and cut out the figures below along the solid lines in Diagram 1 and reassemble them as shown in Diagram 2. The shaded rectangles have different areas: 15 square units in the first diagram and 16 square units in the second diagram. This means that there is an extra square. How can you explain this?

If you would rather not cut out your own shapes, simply compare the two diagrams below.

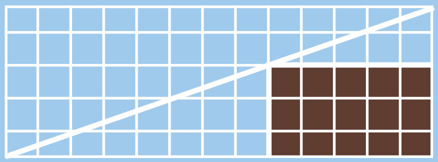


Diagram 1

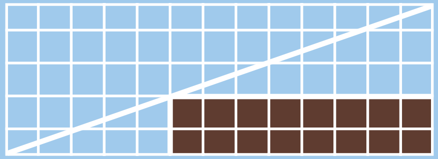
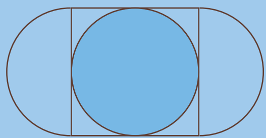


Diagram 2

32

Area

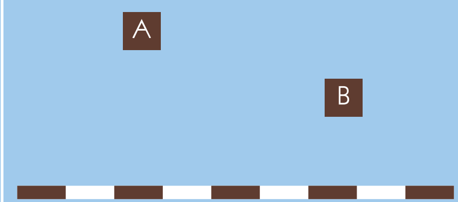
The figure in the diagram below is composed of a square and two semicircles attached as shown in the diagram. Imagine that a circle has been cut out from the square. What is the area of the remaining part?



33

Building a Station 1

Town A and Town B are on one side of a straight railway line but they are located at different distances from it. Where should a railway station be built so that the total transportation costs from the station to each town are at their lowest?

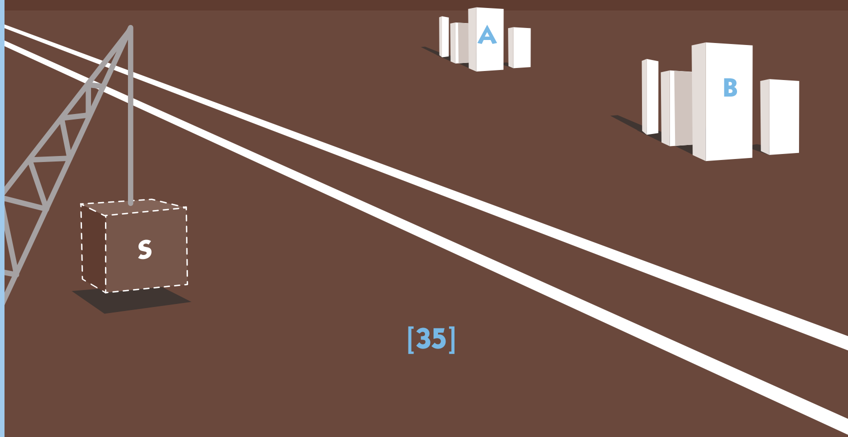


34

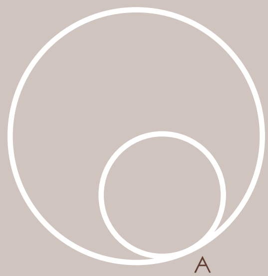
Building a Station 2

Leading on from the previous puzzle, imagine that people living in Town A are not happy with the proposed location of the station because they need to travel further to reach it compared to the residents of Town B. They have suggested that the station be built at a point equidistant from both towns. How can we find the site for the proposed station?

35

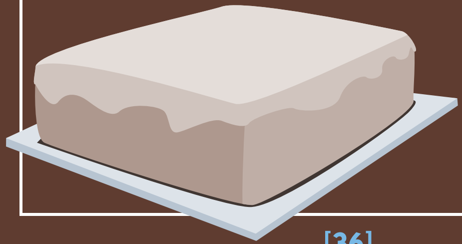


Two Circles
A circular ring rolls without sliding inside another circular ring that has a radius twice that of the inner ring. What will the trajectory of point A be on the rolling ring?



36

Cutting a Cake
Cut a square cake into 4 pieces such that each piece shares a common edge with the other 3 pieces.

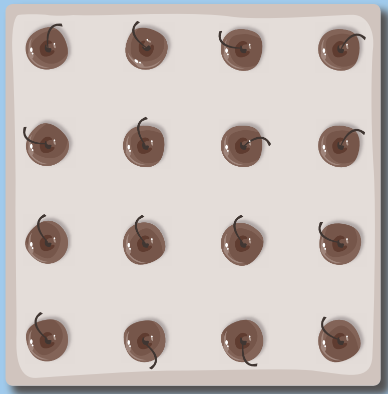


37

Cutting a Square of Paper
Cut a square into:
a) 2 equal pentagons
b) 2 equal hexagons.

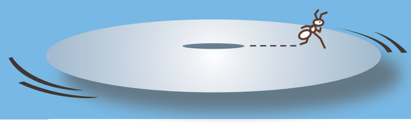
38

Removing Cherries
Remove 6 cherries in the square cake in such a way that there are only 2 or 4 cherries left in each row and column.



39

An Ant on a Disk
A disk rotates at a constant velocity. An ant moves along a radius of the disk at a constant velocity. Draw the path of the ant.



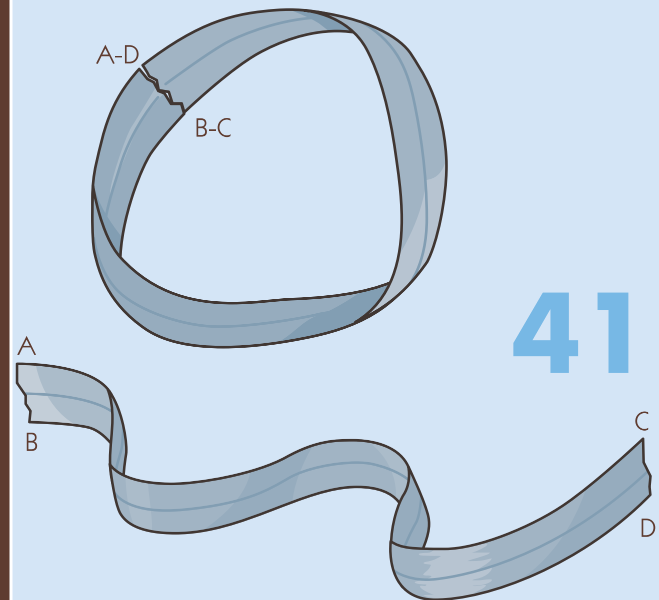
40

An Amazing Strip
This question discusses the Möbius Strip, or Möbius Band, named after August Ferdinand Möbius, a 19th-century German mathematician and astronomer, who was a pioneer in the field of topology.

Begin by cutting out a long rectangular strip of paper (ABCD). Give the strip a half twist and join the ends so that A is matched with D and B is matched with C.

a. Start midway between the 'edges' of your Möbius Strip and draw a line down its centre. Continue the line until you return to (and meet) your starting point. Did you cross any edge? What do you think has happened?
b. Hold the edge of the Möbius Strip against the tip of a felt-tipped highlighter pen. Colour the edge of your Möbius Strip by holding the highlighter still and rotating the Möbius Strip around it. Were you able to colour the entire edge? Why?

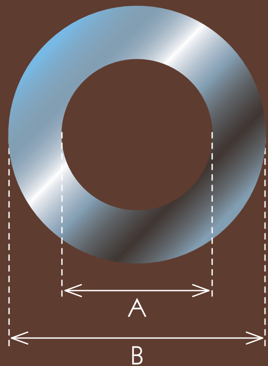
c. Now, using scissors cut your Möbius Strip along the centre-line that you drew in 'a'. If you repeat the process and cut along a new centre-line, what shape do you think you will produce?
d. Can you suggest an application of the Möbius Strip in engineering?



41

Hot Tubes

When most solids are heated, they expand. With this in mind, what happens when a metal tube is heated? Will the diameter of 'A' increase or decrease in size?



42

Solid Glass

When we pick up a glass and fill it with water, we expect the water to stay in the glass until it is poured out. What would happen if a glass tumbler was, in fact, a fluid?

43

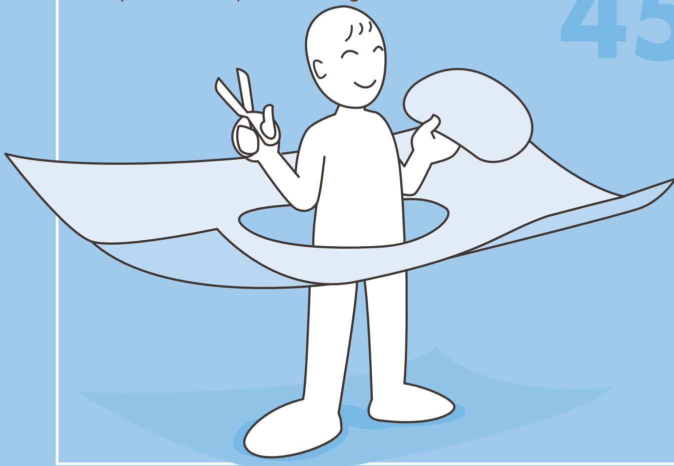
Giant Insects: Science Fiction or Reality?

Insects are the most abundant animals that we observe in day-to-day situations. They are, however, significantly smaller than most mammals. Why are the world's largest animals not insects?

Cutting a Sheet

Using a pair of scissors, cut a hole in an A4-size piece of paper in such a way that you could pass through it.

45



Making It True

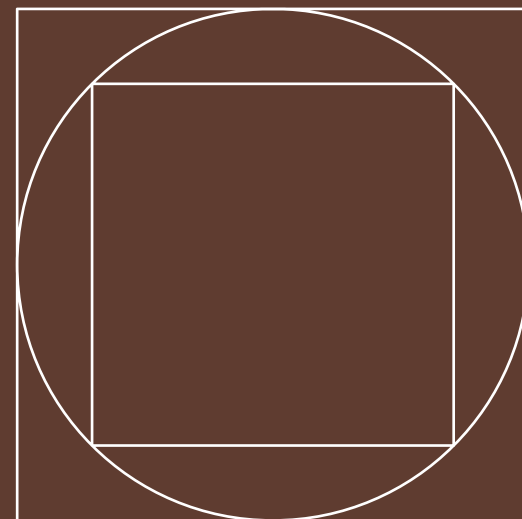
How can you make the equation below true, without correcting anything?

XI + I = X

46

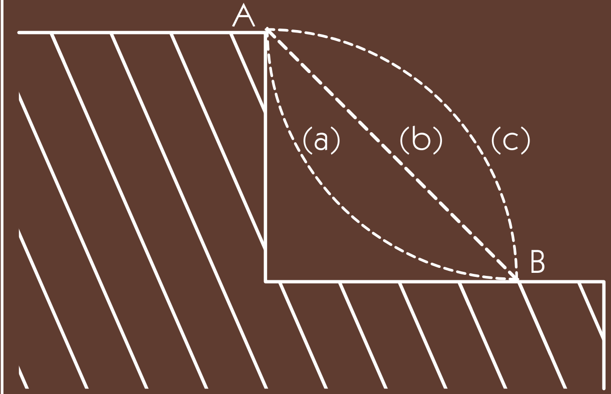
Finding Area

The area of the big square is 100 cm^2. Find the area of the small square without doing any calculations.



47

The 'Fastest' Shape
What should the ideal shape be for designing a slide in a water-theme park such that you would travel from point A to point B in the least time: a), b) or c)?



48

Reflection in a Mirror: A Simple Question
Why do mirrors reverse left and right?

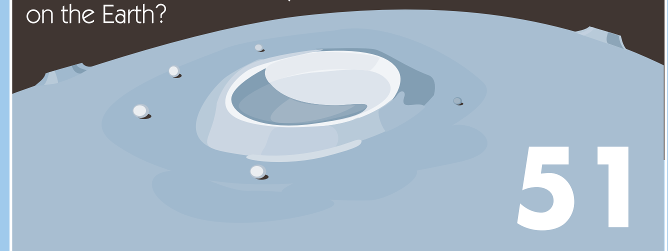
49

Reflection in a Mirror: A Hard Question
Imagine the following: You have a mole on the right side of your lip but when you look in the mirror, your reflection shows the mole on the left side of the lip. If you wave your right hand, your reflection will wave the left hand. This is because the mirror reverses the left and right of our reflected image.

This leads to an obvious question: If a mirror reverses left and right, why doesn't it also reverse top and bottom? For instance, if you face a mirror and pat the top of your head, your image will do the same, and not pat the soles of your feet. Why?

50

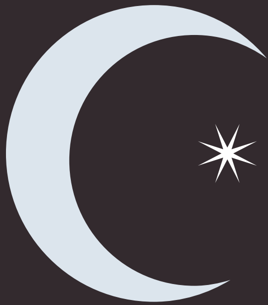
Craters and their Shapes
The images below show fragments of the surface of the Moon. What are the circular structures on the Moon's surface and how did they form? Are there similar structures on the Earth?



51

Shapes and Symbolism

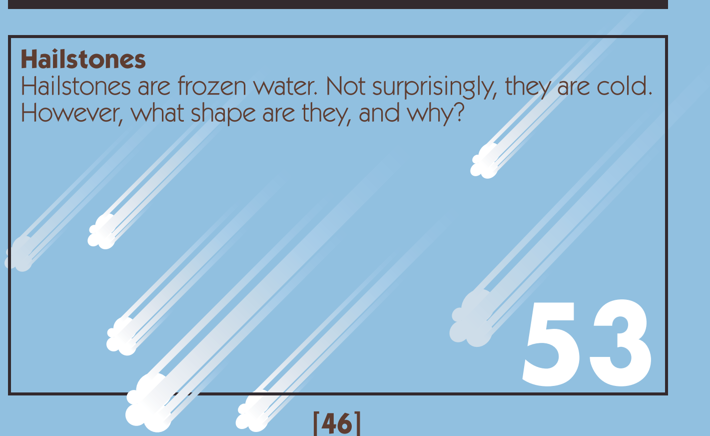
Look at the picture below showing the Moon and a star. Is there anything scientifically unusual about it?



52

Hailstones

Hailstones are frozen water. Not surprisingly, they are cold. However, what shape are they, and why?



53

Desert Glass

For thousands of years, people have been finding strangely shaped pieces of glass scattered throughout parts of the Australian desert. Most of the glass pieces are no more than 25 mm long. However, the shapes include spheres, dumb-bells and even rings. They are made of a very dark, almost black, opaque glass (although if you hold one in front of a bright light, it appears to be dark green). What do you think the origin of the strange shapes might be?

The coin, a euro, is drawn to scale (23 mm in diameter).

54

Art Class

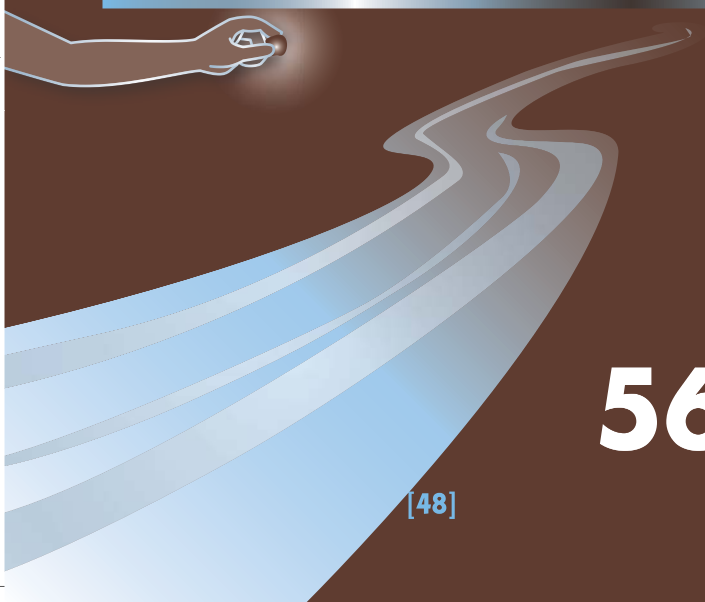
The following picture was proudly drawn by Tommy, an enthusiastic natural landscape artist, in a first-year art class.

Why do you think this could be so?

55

Circles on Water

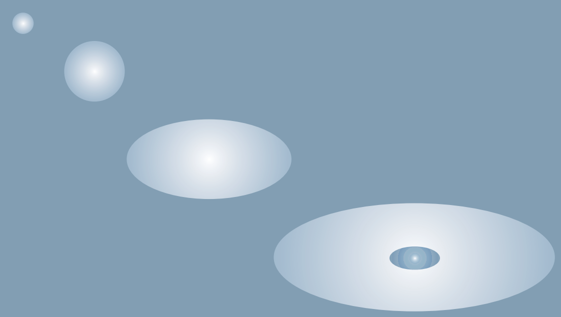
Throwing a stone into still water creates a circular ripple pattern. What will the shape of the ripple be if you throw a stone in a river that has a fast current?



56

Shapes of Stars

We know that stars are hot gaseous spheres. The question is: Which of the objects shown in the figure below could represent stars?



57

A Great Painting

The Italian painter, architect and writer Giorgio Vasari (1511–1574) is best known for his book *Lives of the Most Eminent Painters, Sculptors and Architects*. This is how Vasari describes a story involving the great Italian artist Giotto (1267–1337).

"...Pope Benedict sent one of his courtiers into Tuscany to see what sort of a man he was and what his works were like, for the Pope was planning to have some paintings made in St Peter's. This courtier, on his way to see Giotto and to find out what other masters of painting and mosaic there were in Florence, spoke with many masters in Sienna, and then, having received some drawings from them, he came to Florence. And one morning going into the workshop of Giotto, who was at his labours, he showed him the mind of the Pope, and at last asked him to give him a little drawing to send to his Holiness. Giotto, who was a man of courteous manners, immediately took a sheet of paper,..." and in a couple of seconds drew...something.

Vasari continued:

"...Having done it, he turned smiling to the courtier and said, 'Here is the drawing.' But he [the courtier], thinking he was being laughed at, asked, 'Am I to have no other drawing than this?' 'This is enough and too much,' replied Giotto."

Later, when Giotto's drawing was taken to Rome and explained to the pope, the pope and many of his courtiers understood that Giotto greatly surpassed all the other painters of his time. As a result, the pope invited Giotto to paint in St Peter's.

What did Giotto draw for the pope?

58

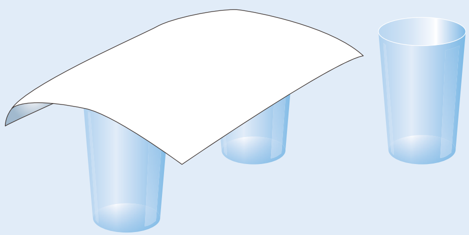
Three Matches

Place 3 matches in the middle of a desk in such a way that their tips do not touch the surface of the desk.

59

Three Glasses

Put 2 glasses on the desk 10–15 cm apart and place a sheet of paper over the tops of the glasses. How should you change the shape of the paper so that it can support a third glass placed in the middle of the paper?

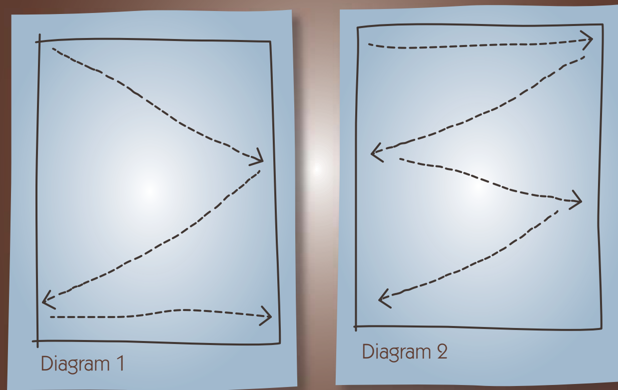


60

Tricky Swim

Two people were watching a fish swimming in a cubic-shaped aquarium from a distance. The first person looked through the front face of the aquarium and drew Diagram 1 showing the movements of the fish.

The second person looked through the right face of the aquarium and drew Diagram 2 showing the movements of the fish.



Using the 2 diagrams above, sketch the movements of the fish as though you were looking at it from above.

61

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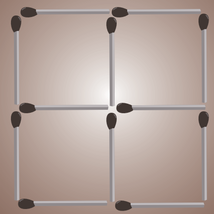
Two Squares

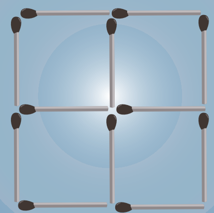
Twelve matches have been used to make the pattern below. Re-arrange them to produce 2 squares by moving only 2 matches.



62

Two Separate Squares

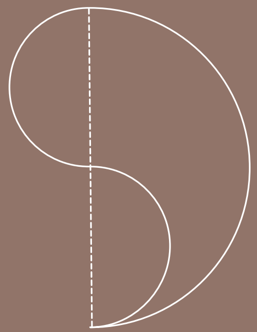
Twelve matches have been used to make the pattern below. Re-arrange them to produce 2 separate squares by moving only 4 matches.



63

Dividing a Symbol

Divide the oriental symbol shown below into 2 identical parts using just one cut.



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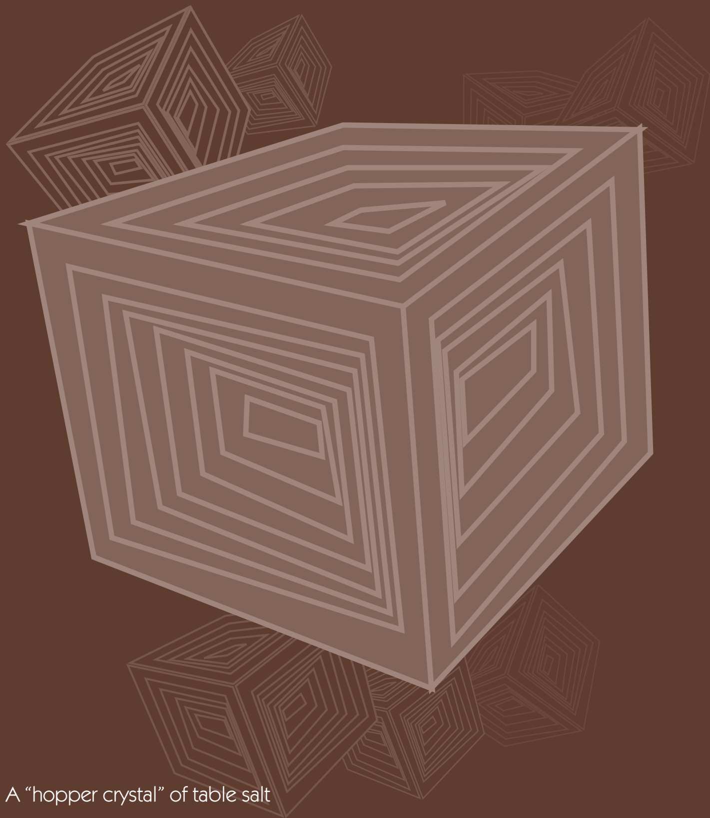


A “hopper crystal” of table salt

THE ANSWERS

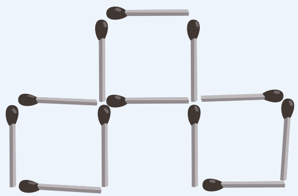
1. A Crystal Shape

Table salt forms crystals that are perfect cubes. This crystal structure is due to the size and proportion of the ions (charged atoms) that make up the crystal. In table salt, there is an equal ratio of sodium and chlorine ions. The most compact way for them to group is as a cube, where each ion of sodium is surrounded by 6 ions of chlorine, and vice versa.

In rare instances, salt forms interesting “hopper crystals”. If you are lucky, you may have such crystals in your sample. Hopper crystals form what may be termed a “crystal skeleton”, in which only the edges extend outwards from the centre of the crystal, leaving hollow stair-step faces between these edges. Hopper crystals form due to the disparity of growth rates between the crystal edges and the crystal faces. Variations in crystal growth rate may arise from pressure and temperature fluctuations of the solute (the saturated salty liquid).

Crystals of materials with fixed formulae, such as table salt, are generally very regular in form.

2. Reducing the Number of Squares
See the diagram below.



3. Leaves
Leaves are designed to capture as much of the Sun's energy as possible and to be as light as possible. In this respect, they are similar to solar panels that supply energy for spacecraft. The most efficient type of leaf is one that is thin and flat. Inside the leaf, exchange of gases and chemical reactions takes place during which carbon dioxide and water are combined to make carbohydrates. The exchange of gases is more efficient if it takes place near the surface of the leaf, close to the surrounding air. A flat leaf, with small holes (stomata) on its underside through which gases may move, is the most efficient design for this purpose.

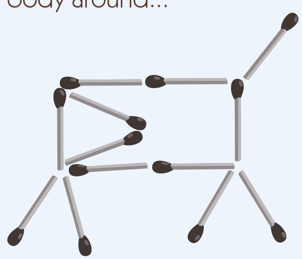
4. Changing the Number
Add the letter 'S' to form the word 'SIX'.

5. Spiral Shape of Galaxies
Spiral arms are seen in about two thirds of the brightest galaxies. We live in a spiral galaxy which we call the Milky Way. If you assume that the arms are made up of stars that always remain in the arms and are just more numerous than the stars outside of the arms, then very quickly the arms would 'wind up' as the galaxy rotates. This does not happen. This is because the spiral arms are, in fact, 'density waves' that move around at a different speed relative to the stars themselves, i.e. the stars move in and out of the spiral arms.

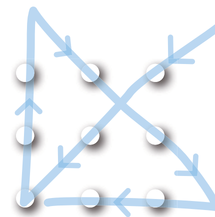
If we simply count all the stars without distinguishing on the basis of brightness, we will find that the density of stars (i.e. the number of stars in the unit of volume of space) in and between the spiral arms is almost the same. The spiral arms look so different from the inter-arm spaces because the spiral arms contain a greater number of hot, bright and young stars. This fits in with the idea of there being more star formation in the arms.

How these density waves are established is unclear, but it may have something to do with interactions between galaxies (many grand design spirals have smaller companions).

6. Changing the Direction
See the diagram below. It was not said that the dog should turn its body around...



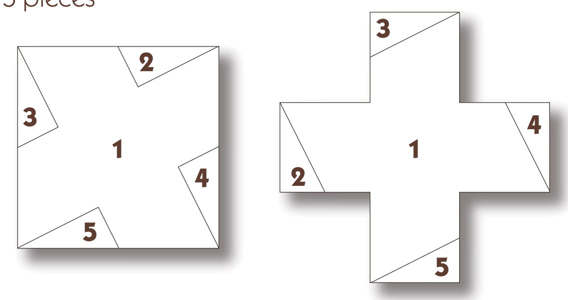
7. Crossing the Dots
See the diagram below.



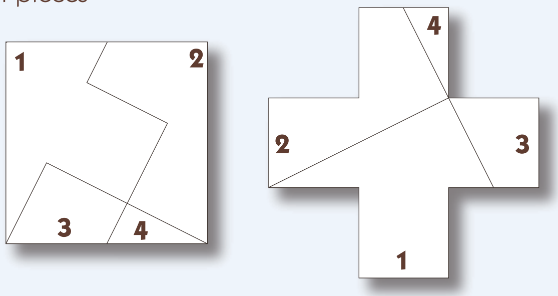
This popular puzzle is often used to illustrate thinking ‘outside the box’ or ‘beyond the boundaries’.

8. Changing the Shape
See the diagrams below.

a. 5 pieces



b. 4 pieces

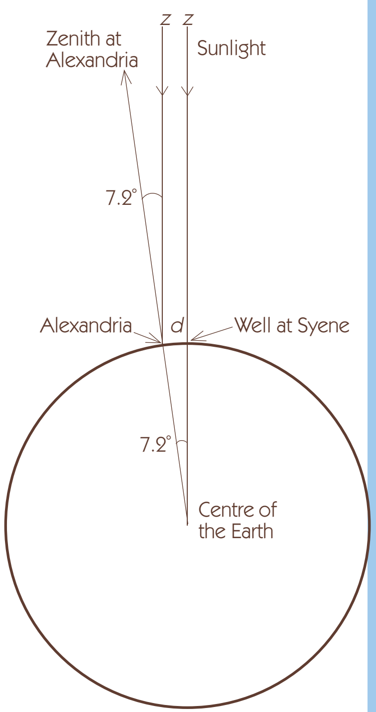


9. Eratosthenes' Problem

The information provided is enough for *you* to solve this problem but was not enough for those in ancient times. In order to solve this problem, Eratosthenes had to make an assumption about *the shape* of the Earth. His great merit was in resolutely stating that the Earth is spherical—a fact that we now take for granted. (In reality, the Earth is not a perfect sphere, it is, in fact, a geoid. It is, however, very close to a sphere and thus is sufficient for these calculations.) Eratosthenes also deduced that the Sun's rays would reach the bottom of the deepest well in Syene when the Sun was exactly in the zenith on Midsummer's Day.

The drawing illustrates this situation geometrically. Here d is the distance between the two locations (Alexandria and Syene) and $z = 7.2°$ is the difference in the elevation

of the Sun in these cities. Now, the proportion that the arc d to the circumference of the Earth L is equal to the proportion of the angle z to the angle 360°, that is $d/L = z°/360°$, whence $L = d \times \frac{360°}{z°} \approx$ 250,000 stadia ≈ 40,000 km. Taking into account that $L = 2\pi R$, where R is the radius of the Earth, we can get $R \approx 6400$ km, which is very close to the modern value of the radius of the Earth.



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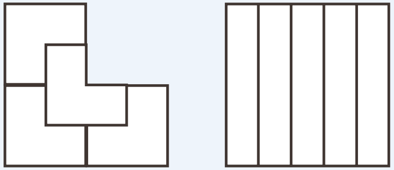
10. Dividing the Figures
See the diagrams below.

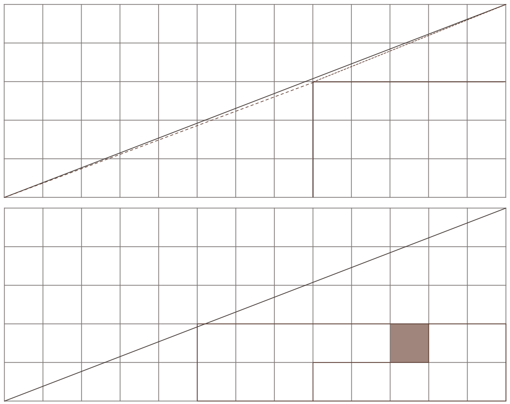


It is an interesting fact that apart from the 5 parallel strips (which must be determined by accurate measuring), there is no other known solution to divide a square into 5 identical parts.

11. A Missing Line
Each of the new 9 lines is slightly longer (by $\frac{1}{9}$) than each of the original 10 lines. That is where the missing line has gone. The difference is not big, which is why it appears that one line has vanished.

12. A Missing Square
The biggest right-angled triangle in the first diagram has legs measuring 5 and 13 units. (The legs are the two sides that make up the right angle in a right-angled triangle.) It looks like one of the two smaller right-angled triangles has legs measuring 3 units and 8 units and the other 2 units and 5 units. However, the construction shown in the first diagram is not possible. How do we prove this?

From the diagram, we can see that all three triangles have the same angles and are therefore similar. That means that the ratio of the lengths of a smaller leg to a bigger leg in each triangle must be the same, that is $\frac{3}{8}$ equals $\frac{2}{5}$ and equals $\frac{5}{13}$. This, however, is not true. As decimals, these ratios are 0.375, 0.4 and 0.385, respectively. As you can see, they differ by only a small amount. This is why we perceive the first diagram to be true. However, if we construct an accurate diagram in which the two smaller triangles have legs measuring 2 by 5 units and 3 by 8 units, we will have a gap along the diagonal. The area of this gap would be the same as the area of the extra square in the second diagram, that is 1 square unit.



13. Snowflakes
Snowflakes are crystals of solid water. When water freezes into ice, the molecules group together in a regular crystalline lattice. In ice this lattice has hexagonal (six-fold) symmetry. It is this hexagonal crystal symmetry that determines the shape of a snowflake. The process begins with a microscopic 'first crystal' which forms as a hexagonal prism. Due to variations in molecular forces, some parts of the crystal (i.e. the facets) will grow faster than others. Water molecules in the air attach themselves to these facets and advance the crystal with its wonderful six-fold symmetry to produce what we know as snowflakes.

Variations in snow crystals are a result of subtle changes in temperature and humidity during the time of crystal growth.

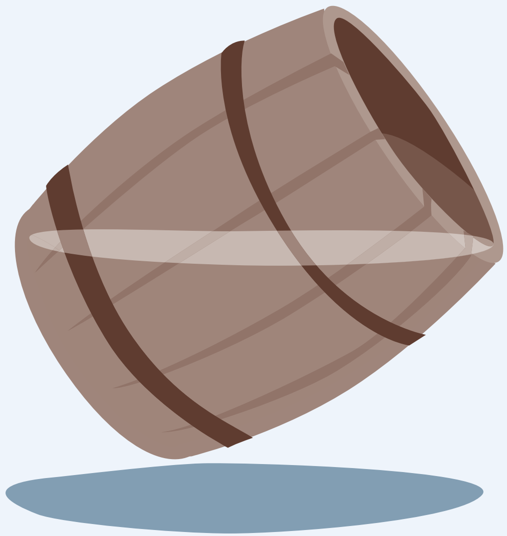
14. Volcanic Shapes
The shape of a volcanic cone is determined by the temperature of the lava and the percentage of gases dissolved in the lava. High-temperature lava with low gas content, such as the basalts of Kilauea, is very fluid and flows away quickly from the vent, producing flattened domes or what are known as *shield volcanoes*. Eruption temperatures of these volcanoes commonly reach 1200° C. Under certain conditions, such lava is sufficiently fluid to flow at speeds ranging from 10 to 30 km per hour. If there is a change in the chemistry of the magma chamber

beneath the vent, such as an influx of volatiles (e.g. water), then the lava generally becomes more viscous, with scoria being produced. The resulting scoria cones can be quite steep. Lower-temperature lava such as the andesites that form the classic *stratovolcanoes* such as Vesuvius (Italy), Mt. St. Helen's (USA) and Mt. Egmont (New Zealand) commonly erupt at temperatures of 900° C. The viscosity of such lava is much higher than basalts, resulting in steeper cones. Lava will not flow at temperatures that fall much below 700° C. As a result, pressure that builds up in magma chambers at temperatures in this range can result in highly explosive eruptions.

15. Dividing the Moon
See the diagram below.

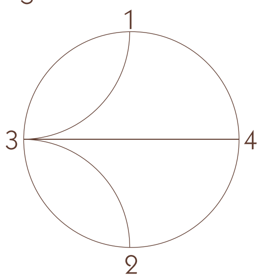
16. Filling a Barrel

Tip the barrel as below and fill it with water until the water touches the opposing edges at the top and the bottom of the barrel.



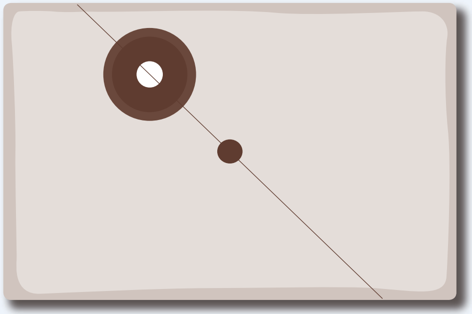
17. Euler's Problem

This problem is equivalent to trying to draw the figure below without lifting your pen from the paper. The vertices 1 and 2 represent the banks of the river, the vertices 3 and 4 represent the islands and the lines represent the bridges.

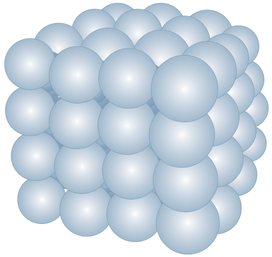


We will use the following general approach. A vertex is called *even* if it has an even number of lines coming to (or leaving from) it. A vertex is called *odd* if it has an odd number of lines coming to (or leaving from) it. You can go through (i.e. come to and leave) any even vertex. If the vertex is odd, you can only start or finish at the vertex. Therefore if a figure has more than two odd vertices, it is impossible to draw it without lifting your pen from the paper. The figure above has 4 odd vertices. That is why it is impossible to draw it without lifting your pen from the paper. It means that the answer to Euler's question is 'No'.

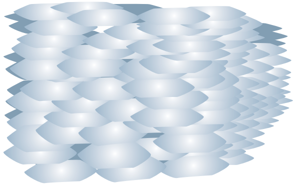
18. Cutting a Cake
If a figure has a centre of symmetry, then any straight line passing through it divides the figure into two equal parts. In this instance, both the rectangle and the circle have a centre of symmetry. The required cut should be along a straight line connecting the two centres of symmetry. See the diagram below.



19. The Nature of Soil
Sand grains generally have an isometric shape, which means that their dimensions approximate a sphere or a cube (although they are very rarely either perfect spheres or cubes). The grains of sand pack up against each other in a way that results in an open porous structure. Consequently, it is not possible to pack spherical grains in such a way that the gaps between the grains are eliminated.



close packed arrangement of isometric (spherical) particles



laminar arrangement of clay particles (sheet-like)

Clays, however, are made up of sheet silicate minerals. These minerals form very flat crystals, rather like sheets of paper. This type of shape allows the clay particles to become tightly packed together so that there are almost no gaps. Therefore almost no water can move through the structure. Rocks that are composed primarily of clays are termed *aquicludes*. Porous, sandy rocks are called *aquifers*.

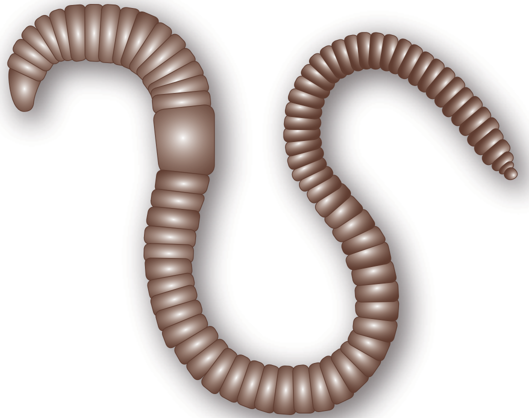
20. Tubular Worms

Tapeworms live in watery environments, such as our intestines (from a tapeworm's perspective, our gut functions primarily as a nutritious soup). Because of the support this fluid provides, they need little support for their bodies. All that tapeworms need in the way of food is obtained through diffusion from the contents of the host's gut. As a consequence, tapeworms do not need to burrow. When mature, eggs are released by the worm and are passed out through the host's gut; many eggs fall upon the grass and are consumed by grass-eating animals. Thus the life cycle continues.

Earthworms, however, have chosen a much less friendly environment: firstly, they are not bathed in nutritious fluids; secondly, they need to burrow through earth that may be fairly compact in order to obtain nutrients. The cross section of an earthworm is circular and is supported by a powerful musculature. Earthworms are able to push their bodies through soil by passing waves of contraction along this musculature. This movement is assisted by stiff external hairs (setae) that are used to anchor (grip) the surrounding soil. Earthworms feed on organic matter in the soil that is ingested in the process of burrowing.

This question is related to the Law of Parsimony, which states that the simplest solution is also the best one. In the above case, we find that the tapeworm does not possess a complex musculature *simply because it is not needed.*

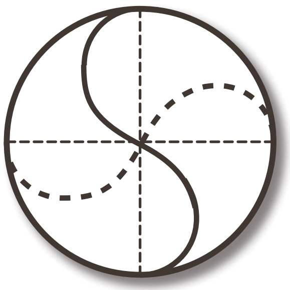
However, we must be a little cautious here as some organs in sophisticated animals, such as the appendix in humans, appear to have no purpose *but they are still present*. In this case, our appendix probably did have an important function in the past (e.g. we may have been herbivores) but as our diet changed, the need for the appendix disappeared. The appendix is thus vestigial.



21. Winning Strategy at the Pizzeria
You need to start by placing a slice of salami in the centre of the pizza. After that you must always place your slice directly opposite the slice placed by your friend, i.e. symmetrically about the centre of the pizza.

22. Animal Size
On the Earth, the primary limiting factor concerning the size of multicellular animals is gravity. If animals are to move, they must be able to lift and move their limbs. Although the force of gravity acts on all objects on the Earth's surface, its effect is reduced by the buoyancy that any surrounding medium, such as water or air, provides. The density of animals is proportionately much greater than air (at standard pressure and temperature, the density of air is about 0.0013 grams per millilitre while that of animals is a little over 1 gram per millilitre). Because of this, water with a density of 1 gram per millilitre provides greater buoyancy than air. The force required to support land animals is several orders of magnitude greater than in (or on) water. Further, as the size of a land animal increases, the percentage of its skeleton that forms part of its body mass increases; this places a limit on very large terrestrial animals. The effects of gravity, and thus restrictions on size, are reduced in water.

23. Dividing Yin and Yang
See the diagram below. (*Note:* the dotted vertical and horizontal lines are construction lines.)



24. A Berry in a Wine Glass
See the diagram below.

25. Two Containers

Because the containers have the same shape, all their dimensions are proportional. As the volume of the big container is 8 times greater than the volume of the small container, it means that both the radius of the base and the height of the big container are twice those of the small container. Therefore, the surface area of the big container is 4 times larger than the surface area of the small container. This means that the weight is too as both containers are made of the same material and have the same thickness.

It can be shown using simple mathematics. Let r be the radius of the base and h the height of the small container. Because the dimensions of the 2 containers are proportional, the big container has the radius kr and height kh. From the relationship between their volumes, we can find k:

$$\frac{V_1}{V_2} = \frac{\pi (kr)^2 \, kh}{\pi r^2 h} = 8 \Rightarrow k^3 = 8 \Rightarrow k = 2$$

The relationship between the surface areas is:

$$\frac{S_1}{S_2} = \frac{\pi (2r)^2 + 2\pi \times 2r \times 2h}{\pi r^2 + 2\pi rh} = \frac{4\pi r (r + 2h)}{\pi r (r + 2h)} = 4$$

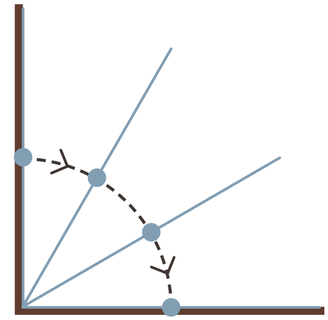
The relationship between the weights is:

$$\frac{w_1}{w_2} = \frac{m_1 g}{m_2 g} = \frac{m_1}{m_2} = \frac{v_1 p}{v_2 p} = \frac{v_1}{v_2} = \frac{S_1 T}{S_2 T} = \frac{S_1}{S_2} = 4$$

where m_1 and m_2 are the masses, g is the acceleration due to gravity, v_1 and v_2 are the volumes of the material, p is the density of the material and T is the thickness. So the big container is 4 times heavier than the small container.

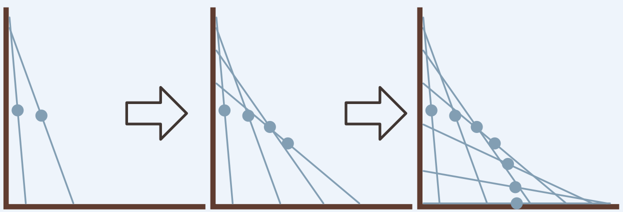
26. A Cat on a Ladder

Part 1. Most people are confident that C is the answer to Part 1. Without much difficulty, you can imagine the ladder rotating about a centre—where the base of the ladder touches the wall. An arc is the result and it represents a quarter of the circumference of a circle.

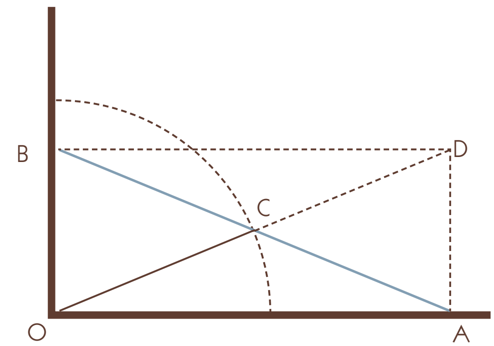


Part 2. However, Part 2 tends to be more problematic, with many people concluding that A is the correct answer. It seems logical to many of us that as the ladder slides outwards away from the wall, the ladder will appear to drop quickly, then level out as the ladder approaches the ground. However, the answer to Part 2 is also C.

Check it out by making a model (see the sequential sketches below). Using a paper ladder with a point marking half way, slowly slide the ladder down and away from the wall. After each small movement, mark a dot on the page at the place where the centre of the ladder lies. Note that as the ladder approaches the ground, further lateral movement is minimal.



Simple proof is below:

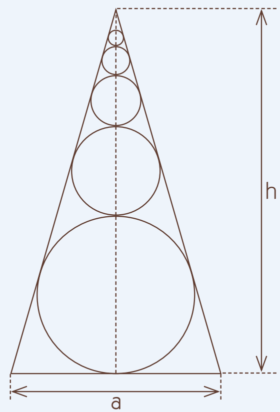


Let AB be the ladder. Point C is where the cat is sitting and is always the same distance (i.e. half of the length of the ladder) from point O regardless of the position of the ladder. This comes from the fact that diagonals in a rectangle are the same and are divided in halves by the point of their intersection.

You may be surprised to see that the trajectory is the same in both cases. Do not be alarmed, however. In this case, the intuition of many people fails. We tried this test out with a class of 100 4th-year engineering students in Australia, Germany, New Zealand and Norway. These young men and women, aged about 21, are expected to be able to quickly conceptualise shapes, dimensions, movements and forces. The students were given 40 seconds to answer the puzzle, with instructions that it was a *mental* exercise, i.e. no calculations or drawings were allowed. The results were startling. Although 74% of the students gave the correct answer for Part 1, 86% gave the wrong answer for Part 2. Of all the students, 14% and 34% gave B as the answer for Parts 1 and 2, respectively.

27. Circles

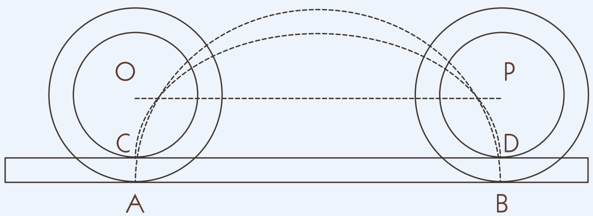
If you look at the diagram below, you will see that the sum of the diameters of the infinitely many circles inside the triangle equals *h*.



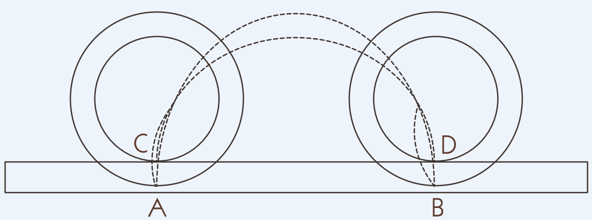
28. Wheels

It is impossible for both wheels to roll. Only the big wheel can roll. When it does, the small wheel, apart from rotating, also slides on the surface of the rail. When the big wheel makes one rotation and covers distance AB, then point C moves to point D. Distance AB equals the circumference of the big wheel. Obviously AB = CD. But CD is bigger than the circumference of the small wheel because the small wheel, apart from making one rotation, also slides on the surface of the rail.

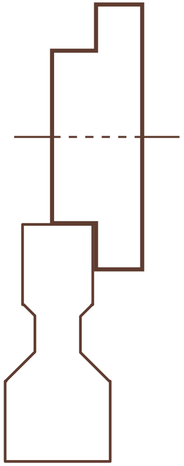
Point A on the big wheel (which rolls) traces a curve called a cycloid. Point C on the small wheel traces a curve which looks like a flattened cycloid. If the radius of the small wheel is very small—almost zero—then the trajectory of point C would be very close to the straight line OP. In this case, the small wheel would mostly slide because the distance CD would be much bigger than the length of the circumference of the small wheel. It is very unlikely that such construction would be viable because the sliding of the small wheel would create permanent friction.



The small wheel can roll only if the big wheel does not come into contact with the surface. Point C on the small wheel would trace a cycloid and point A on the big wheel would trace a curve that resembles an 'unfinished circumference'. If the radius of the small wheel is very small—almost zero—then the trajectory of point A would be very close to the circumference of the big wheel. The lowest point on the big wheel for some moments would actually move in the opposite direction to that of the overall movement. There would be a small loop on the bottom of its trajectory.



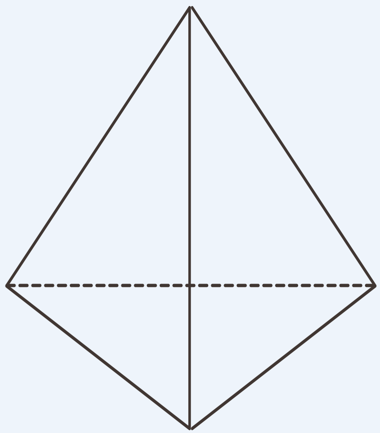
This is how real railway wheels move; the small wheel rolls and the big wheel does not touch the surface. See the diagram below, showing a section through the wheel and the rail.



29. Counting Triangles
Careful counting gives you 35 triangles.

30. Counting Squares
Careful counting gives you 45 squares. Don't forget about the squares that are positioned obliquely.

31. Constructing Triangles
It is impossible to do on the plane but in three-dimensional space a right triangular pyramid has 4 equilateral triangles. See the diagram below.

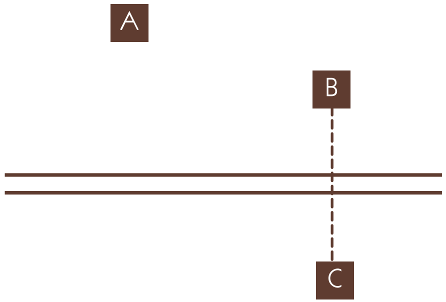


32. An Extra Square
The explanation is very similar to the answer for puzzle 12.

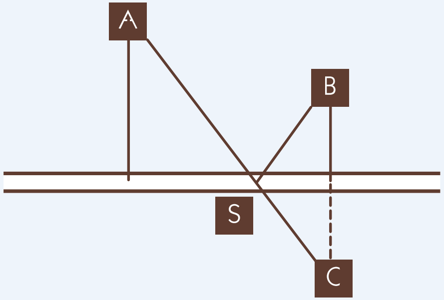
33. Area
The area is 100 cm^2. Imagine that you move the two attached semicircles by 5 cm towards the centre of the circular hole. They will fill the hole, reforming the square that has sides of 10 cm and giving an area of 100 cm^2.

34. Building a Station 1
The total transportation costs from the station to each town are at their lowest if the total distance from the station to each town is at its shortest.

To begin, one of the towns should be reflected on the other side of the railway line. Let's say we reflect Town B to create point C.



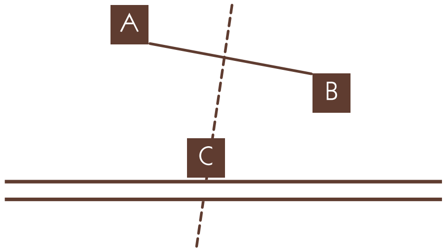
By connecting the points A and C, S, a point along the railway line, will be created.



The shortest distance between two points is a straight line. In this case, the line AC would be the shortest distance from A to B through S. The reason being that SB = SC, since C is a reflection of B. Therefore, S is where the station should be built.

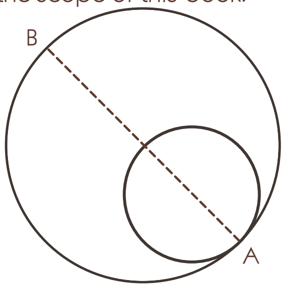
35. Building a Station 2

In this case, you should connect the points A and B and draw a straight line perpendicular to the line segment AB through its midpoint. All points along that line are equidistant from A and B. The intersection of that line and the railway is point C. This is where the station should be built.



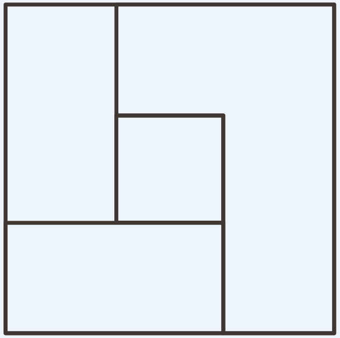
36. Two Circles

The answer to this puzzle is quite unexpected for many people. You may find it easier to visualise if you demonstrate using 2 rings. Point A will move along a straight line. After the inner ring does one rotation, point A will move to point B and then return to its initial position. It takes some facts from geometry to prove this fact but this is beyond the scope of this book.



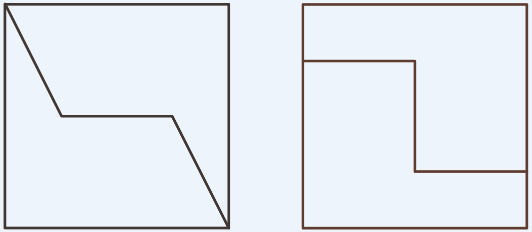
37. Cutting a Cake

Below is one possibility.



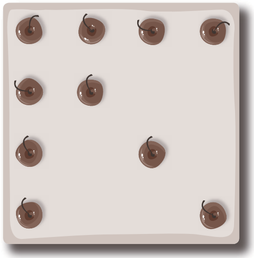
38. Cutting a Square of Paper

Possible answers are below:



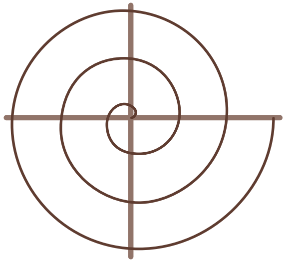
39. Removing Cherries

One of the possible answers is below:



40. An Ant on a Disk

The path of the ant will be a curve that is known as Archimedes' Spiral, as illustrated below.



41. An Amazing Strip

a. Möbius strips have found a number of surprising applications that exploit a remarkable property they possess: one-sidedness. If you joined A to C and B to D (without doing a half twist), you would produce a simple belt-shaped loop with two sides and two edges, making it impossible to travel from one side to the other without crossing an edge. But as a result of the half twist, *the Möbius Strip has only one side*!

b. Yes. If you did it correctly, you would have been able to colour the entire edge. This is due to another amazing property of the Möbius Strip—*it has just one edge*!

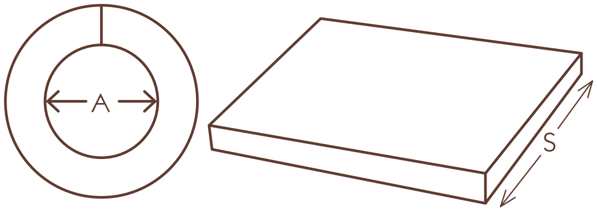
c. When you first cut a Möbius strip, you get a strip that is twice as long as the initial strip. But it is no longer a Möbius Strip. By drawing the centre line, you can see that this new strip has one side and one edge. When cutting it for the second time, you get 2 strips that cannot be separated. These are not Möbius strips either.

d. As far as use is concerned, giant Möbius Strips have been used as conveyor belts (to make them last longer since 'each side' gets the same amount of wear) and as continuous-loop recording tapes (to double the playing time).

42. Hot Tubes

The question may be re-phrased to the following: Does the hole diameter ('A') get bigger or smaller?"

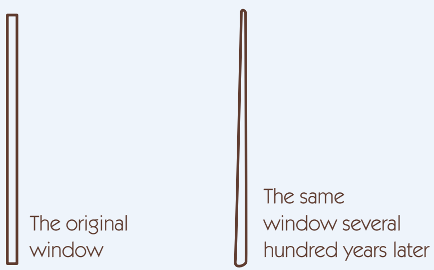
The best way to work this out is to cut the tube (longitudinally) and roll it out flat. The resulting shape would be something like this:



If we heat the tablet-shaped plate on the right, expansion will be uniform throughout it. Because of this, the change in dimension will be greatest along the length of the pipe(s) but it will also occur at the base of the plate (i.e. the outside of the pipe) and the top of the plate (i.e. inside the pipe). The differences between these two are generally not significant unless the pipe is very thick. Thus expansion will occur in a pipe such that the inner and the outer surfaces will both increase. If you mentally roll the plate back into the shape of a pipe, you will see that this will result in an increase in the diameter of the hole, i.e. 'A' becomes longer.

43. Solid Glass

Not much would be different as glass is, in fact, a fluid; in this case a supercooled liquid in which molecules are arranged in a disordered fashion, much as you would expect in a gas. The distinction between glass and what we call 'liquid' is solely on the basis of viscosity. Glass does flow, as can be seen with old windows, which become much thinner at the top. However, the process is very slow.

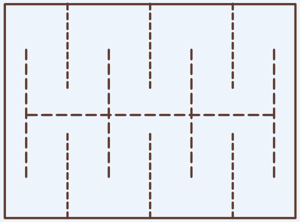


44. Giant Insects: Science Fiction or Reality?

Insects are characterised by a rigid exoskeleton. All muscles must attach themselves to this skeleton to permit movement. To stop the muscles from collapsing the exoskeleton, the skeleton must be both strong and relatively rigid. As the animal grows in size, the strength and the mass of the exoskeleton must increase, too. This is not a linear relationship for as the body gets bigger, the thickness of the exoskeleton increases disproportionately. This is especially the case for insects living in the air. If size increases, eventually the thickness of the walls would be so great that the musculature required to move the animal would not fit inside it. Note that if the insect lived in water, it would be able to maintain a greater size due to the lesser effects of gravity, a result of the buoyancy provided by the surrounding water. (See *also* puzzle 22).

A further complication may be moulting. In order to grow, insects must shed their exoskeleton and grow a new, larger one to fit their increased size. Any new exoskeleton will be soft for a period, making the animal very vulnerable. If the animal was large and lived in the air, it would mean that special measures would need to be taken to protect it from predation whilst the new exoskeleton hardened. A high susceptibility to predation is an obvious major inhibition to large size.

45. Cutting a Sheet
Below is one possible solution.

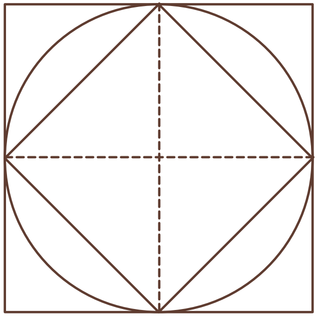


46. Making It True
The equation XI + I = X is written using Roman numerals.

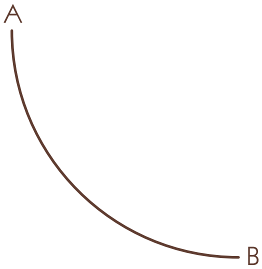
One way to make the equation true is to simply look at it in a mirror. You will see the reflection X = I + IX, which is a true equation (i.e. 10 = 1 + 9).

Alternatively, you can invert the page to get the true equation X = I + IX.

47. Finding Area
The area of the small square is 50 cm^2. To get this answer, rotate the small square as shown in the diagram and you will see that the small square comprises 4 triangles whereas the big square comprises 8 such triangles.

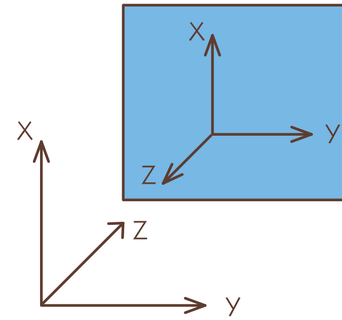


48. The 'Fastest' Shape
The correct answer is (a). The Swiss mathematician John Bernoulli proved in the 17th century that the shape should be an upside down arc of a cycloid. (This was discussed in puzzle 28.) This curve is called a brachistochrone. The proof of this fact is beyond the scope of this book.

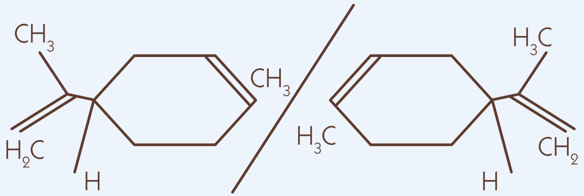


49. Reflection in a Mirror: A Simple Question
The simple answer is that they don't. Look in a mirror and wave your right hand. On which side of the mirror is the hand that waved? The right side. But look again. Mirrors *do* reverse in and out. Imagine holding an arrow in your hand. If you point it up, it will point up in the mirror. If you point it to the left, it will point to the left in the mirror. But if you point it towards the mirror, it will point back at you. In and out are reversed.

If you take a three-dimensional rectangular co-ordinate system, (X,Y,Z) and point the Z axis such that the vector equation $X \times Y = Z$ is satisfied, then the co-ordinate system is said to be right-handed. Imagine Z pointing towards the mirror. In the reflection, X and Y are unchanged but Z will point back at you (see the figure on the next page). In the mirror, $X \times Y = -Z$. The image contains a left-handed co-ordinate system. Mathematically, a mirror changes a right-handed co-ordinate system into a left-handed system.



This has an important effect, familiar mostly to chemists and physicists. It changes the chirality, or handedness, of objects viewed in the mirror. Your left hand looks like a right hand while your right hand looks like a left hand. Molecules often come in pairs called stereoisomers, which differ not in the sequence or number of atoms but only in that one is the mirror image of the other so that no rotation or stretching can turn one into the other. Mirror molecules have different properties. For example, orange and lemon peel both contain a molecule called limonene. They are mirror images of each other but produce remarkably different smells!



(+) Limonene

ORANGE

(-) Limonene

LEMON

The types of limonene in oranges and lemons are mirror molecules (see the figure above). The molecule in the orange is ‘left-handed’ and the one in the lemon is the ‘right-handed’ version.

50. Reflection in a Mirror: A Hard Question

Let us conduct a two-part experiment to find out why. Imagine a camera placed in front of a mirror. Let the camera take a picture of itself. Now compare the camera and its image by placing them side by side. You will see that the mirror reversed left and right, just as expected. (Figure 1.) Note that in order to compare the camera with its image, we had to turn the camera around the vertical axis.

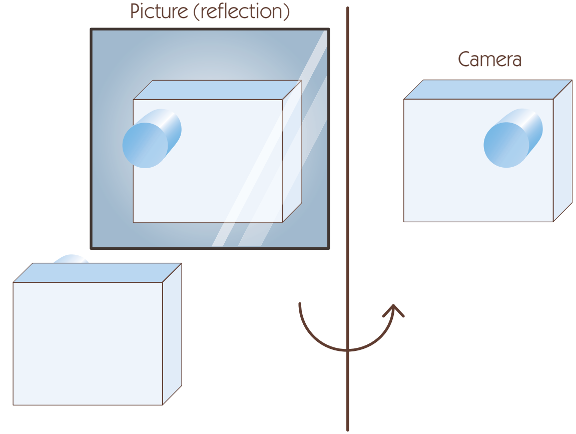


Figure 1

Why did we choose the vertical axis, and not the horizontal or any other axis? What will happen if we choose the horizontal axis? In other words, what if we turn the camera around the horizontal axis? The result is presented in Figure 2.

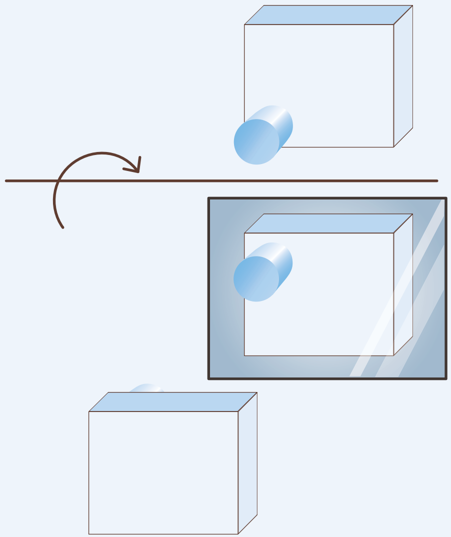
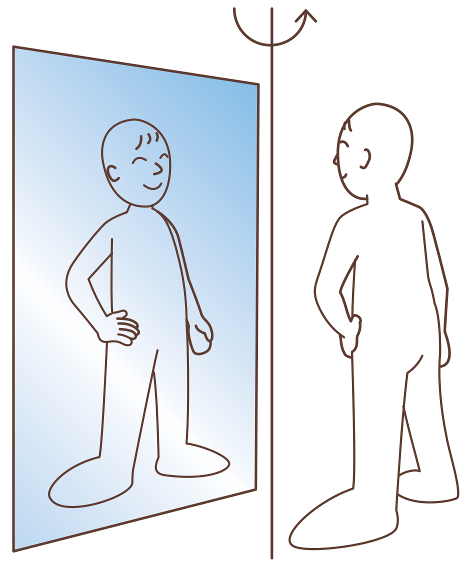


Figure 2

Comparing the camera with its image, we can see that the top and bottom are reversed in this case.

From the two experiments, you can see that a mirror reverses images in different ways, and that the type

of reversal depends on the direction of the imaginary rotation axis. Certainly, it is more natural for us to turn around the vertical axis than around the horizontal axis. That is why we deal with left-right reversal when looking in a mirror



51. Craters and their Shapes
The circular structures are impact craters. These craters are the remains of collisions between an asteroid, comet or meteorite and the Moon. These objects hit the Moon at a wide range of speeds but average about 20 kilometres per second.

The surface of the Moon is scarred with *millions* of craters. Most of the craters on the Moon are circular in shape. Only a few are not circular. Non-circular craters are still an enigma. Scientists do not know exactly how these oddly shaped craters were formed.

The Earth's atmosphere protects the surface of our planet from bombardment from most potential impactors: most objects from space burn up in the Earth's atmosphere. Even if an asteroid or meteorite survives in the atmosphere, hits the Earth and forms a crater, erosion (the action of wind or water) quickly wears away these craters. That is why there are only about 160 identified impact craters on the surface of the Earth today.

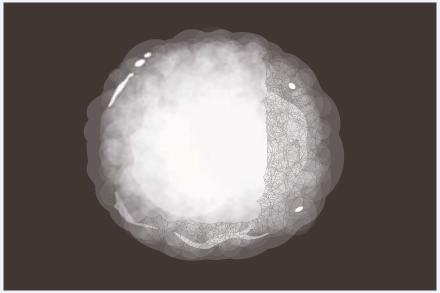
52. Shapes and Symbolism
The Moon is, of course, a big sphere. The bright part is in the sunlight while the dark part is in shadow. However, using logic, the star in this figure is in front of the shadowed portion of the Moon. As the Moon is much closer to us than any other star or planet, no star could ever appear where this one does. You would never be able to see a

star 'inside' the Moon. It will always have to be behind the Moon!

A number of national flags include this Moon and star design. Most flags, however, are generally symbolic rather than scientifically accurate. For example, stars in the heavens are never in groups of 50, all lined up in rows.

53. Hailstones
Hailstones form near the top of thunderstorm clouds from super-cooled rain droplets, when the temperature drops well below freezing point (e.g. –50 to –60° C). However, as you may have observed, hailstones may be many times larger than a raindrop, the size of which is controlled primarily by the surface tension effects of the water droplet. After the original formation, hailstones grow much in the same manner as a snowball grows—they are made up of concentric shells of ice, with the maximum size depending upon the length of time the hailstones are in the air, the availability of moisture and the temperature. If the formation is high in the sky and there are strong updraughts keeping the hailstones in the cold air, hailstones can easily reach diameters of up to 65 mm. Eventually they grow too large to be supported by the updraught and fall to the ground. One of the largest hailstones recorded fell at Chilches, in eastern Spain, in 2000. Here, hailstones were found to weigh up to four kilogrammes, with diameters of 260 mm!

Shapes of hailstones vary but, like snowballs, they are commonly spherical to subspherical.



54. Desert Glass

These small pieces of glass are known as australites and were produced when a meteorite or comet crashed into the Earth some 15,000 years ago. The site of impact is considered to be in an area in the northeast of Thailand. The energy of the impact melted both the meteorite and many of the rocks in the area, throwing these into the atmosphere as globules. The largest globules fell nearest the impact site in Indochina, the smaller globules, 10–20 mm in diameter, were thrown high enough to fall in Australia, thousands of miles to the southeast. The strange shapes are derived from the glassy globules freezing in the upper atmosphere, then partially remelting when falling again. The ring-shaped australites are perhaps the most interesting as they develop from the melted front of the australite, flowing 'uniformly' backwards over the glassy sphere (see figure below). Upon hitting the ground, this ring of glass often became detached from the sphere, producing the almost perfect ring shown in the picture with the puzzle.

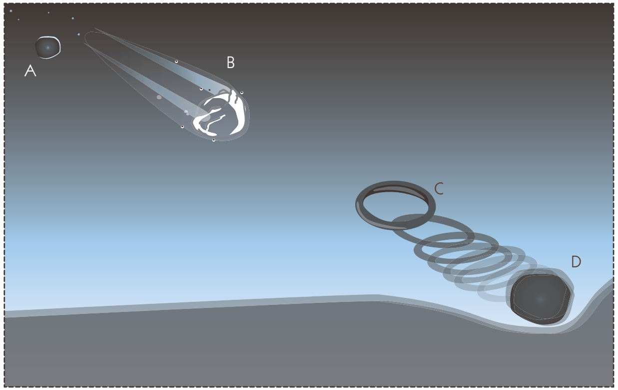


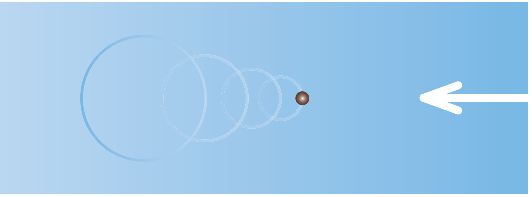
Figure. Formation of ring-shaped glass objects. The original australite (A) enters the lower atmosphere and begins to melt at the leading edge, with glass flowing towards the rear (B). When the glass bead hits the ground, the shock of the impact causes the ring (C) and the inner plug (D) to dissociate.

55. Art Class
The problem with Tommy's picture is reality. Our first instinct is to that the Sun and Moon should not appear in the sky at the same time. Although this is generally the case, they can sometimes be seen together, although the Moon is generally much fainter. Nonetheless, although the shadows from the trees are approximately where we would expect them to be, the shadow on the Moon is not. It should be on the opposite side of the Moon, facing away from the Sun. Finally, consider the clouds. The darker areas of the clouds are actually shadows. The darker the shadow, the denser the cloud must be. However, as the sun provides the light to produce shadows, the shadow in the larger cloud is on the wrong side. It should be facing away from the Sun. (Incidentally, Tommy did not do too badly with the smaller, higher cloud as the shadow is about where we could expect them to be.)

56. Circles on Water
The shape of the ripple will still be circular. Imagine you are in a boat that is floating with the river current. From within the boat, the river seems looks immovable (Galileo's principle of relativity). Therefore, the shape of the pattern will be the same as in immovable water, i.e. a circle. If you threw the stone from the riverbank, you would observe a circle, widening, but moving with the flow of the river. In the figure on the next page, the place where the stone enters the water is shown as a black dot.

The circles of growing diameter illustrate the ripple effect in the water as the river flows.

However, other factors may alter the ripple shape, such as friction. When the water touches the banks of the river it is slowed down. In a narrow stream where the central part flows the fastest, ripples will have an elongated shape.



57. Shapes of Stars
Actually, all of them could represent stars!

The smallest sphere could be a star similar to the Sun.

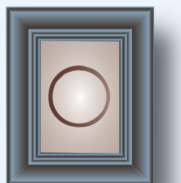
The big blue sphere could be a blue giant star like Sirius.

The big blue oval-shaped object could represent the star Regulus, the brightest star in the constellation Leo. Astronomers found that its rapid spin flattened the star into a shape rather like that of a pumpkin. This massive star has five times the diameter of the Sun, yet it completes a rotation in only 16 hours (the Sun takes a month to rotate).

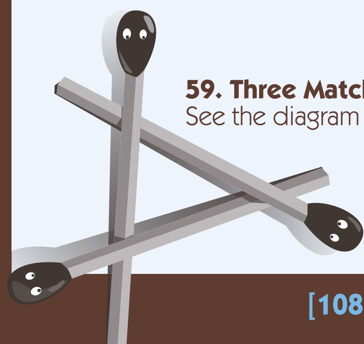
This extreme speed gives Regulus a bulging waistline; in fact, it is spinning at 86% of its break-up speed. If it spun any faster, the star would actually tear itself apart!

The last oval could be an image of a black hole surrounded by the rotating disc of hot matter, falling into the black hole (known as an accretion disc). The black hole marks the final stage in the stellar evolution. We cannot see the black hole as no radiation comes from it. However, matter in orbit around the black hole, in the form of the accretion disc, will gradually be pulled into the hole. As matter gets closer, its rotation speed increases, it gets hotter and the matter then begins to emit light.

58. A Great Painting
Giotto drew a circle, which, according to Vasari was "so perfect that it was marvel to see". He did it without any aid such as a compass, using just one free movement of his hand.

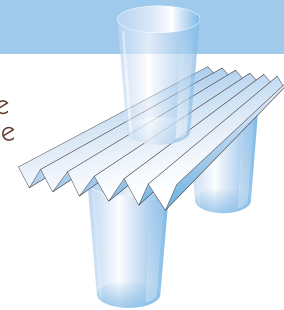


Giotto (1267–1337)

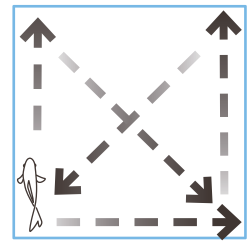


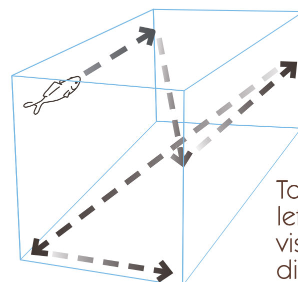
59. Three Matches
See the diagram on the left.

60. Three Glasses
You need to pleat the paper as shown in the diagram on the right.



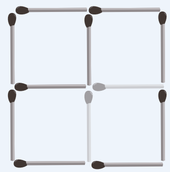
61. Tricky Swim
The view from above is shown in the diagram on the right.



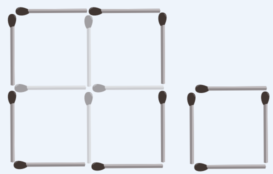


To draw the diagram on the left, you can use your spatial vision or first sketch a three-dimensional diagram of the fish's actual movements in the aquarium.

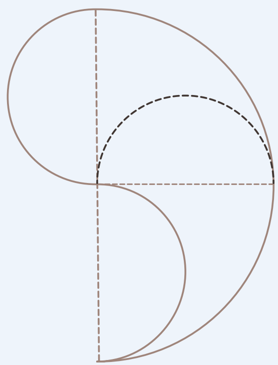
62. Two Squares
See the diagram on the right.



63. Two Separate Squares
See the diagram on the right.



64. Dividing a Symbol
You need to cut along the dotted curve as shown in the diagram on the right.



ABOUT THE AUTHORS

Professor John Buckeridge is Professor of Natural Resources Engineering at RMIT University, Melbourne, Australia and Honorarprofessor at Wismar University of Business Technology and Design, Germany. His research interests involve systems and sustainability engineering and marine biology and geology. He has contributed to more than 170 publications and has been Visiting Professor in many countries. He is currently President of the International Society of Zoological Sciences and chairs the International Union of Biological Sciences Bioethics Committee. He is a Fellow of the Institution of Professional Engineers, New Zealand and served on their Standing Committee on the Environment for 5 years.

Professor Sergei Gulyaev is Director of the Centre for Radiophysics and Space Research at the Auckland University of Technology, New Zealand. He graduated from Moscow State University and worked at the Ural State University (Russia) as Head of Department of Astronomy and Geodesy. His research interests involve astrophysics, earth and planetary science, informatics and statistics. He has written more than 100 scientific publications, including the textbook *Basics of Natural Science: History of the Earth* (3rd edition, 2000).

Dr Sergiy Klymchuk is an Associate Professor of the School of Mathematical Sciences at the Auckland

University of Technology, New Zealand. He has 25 years' experience teaching university mathematics in different countries. His recent research interests are in mathematics education. He has more than 100 publications including several books on popular mathematics and science that have been or are being published in 9 countries: New Zealand, USA, Germany, Greece, Spain, Poland, Singapore, Korea and China.