Teaching Mathematics at University Level

• Considerable research over 30 years into teaching mathematics at school level (PME 1990)
• Research into teaching at university level less well developed
  – Professional literature
  – Pedagogical literature
  – Research literature

What can we learn from the literature? – Treffert-Thomas & Jaworski, 2015
Scarcity of research on teaching practice

• ... while some mathematicians have written about their teaching,

• others have analyzed aspects of their teaching and their students’ learning in innovative collegiate courses,

• and a diverse body of other scholarship mentions collegiate mathematics teaching,

• very little research has focused directly on teaching practice—what teachers do and think daily, in class and out, as they perform their teaching work.

Developmental research

Developmental research is research which has the intention not only to chart, monitor, or evaluate the developmental process, but also to contribute to that development (Jaworski, 2003).

[It is] research which both studies the developmental process and, simultaneously, promotes development through engagement and questioning. ...

(Jaworski & Goodchild, 2006, p. 353).
MEC Projects – 3 examples

1. The Linear Algebra Project
2. Collaborative learning in Mathematical Modelling
3. Second-Year Mathematics Beyond Lectures (The SYMBoL Project)
1. The Linear Algebra Project

Focus on lecturer’s teaching approach

• A collaborative study between 2 mathematics educator-researchers and one mathematician-lecturer teaching a first year module in linear algebra
• Intensive discussion/reflection and lecture observation over one semester
• Focus on lecturer’s actions and goals and their relation to student learning

Collaborative research

Co-learning inquiry: small community of inquiry

• Trusting relationship – bringing mathematics and mathematics education closer together
• Mathematician reflecting using language of linear algebra and his goals in teaching LA
• Mathematics educators tentatively introducing educational terms (e.g., enculturation)
• Developing comfortable ways of speaking to each other.
• Some data from students.
Expository and didactic modes of reflection

Lecturer’s words in a research meeting:

*Thursday is about defining the characteristic polynomial, understanding that its zeroes are the eigenvalues, and I’ll show an example of an eigenvalue that has algebraic and geometric multiplicity 2. (Didactic – actions of teaching)*

Algebraic multiplicity, meaning this is the power with which the factor lamda minus eigenvalue appears in the characteristic polynomial, and geometric multiplicity is the number of linearly independent eigenvectors. (Conveying mathematical meaning)

*And these are the important concepts for determining if a matrix is diagonalisable because, for that, we need sufficiently many linearly independent eigenvectors. (Didactic – goals of teaching)*
Now if an eigenvalue has algebraic multiplicity larger than 1, that means there are correspondingly fewer eigenvalues. So, in principle, we can fail to find as many eigenvectors as we need in that case. On the other hand, if an eigenvector has algebraic multiplicity 3, the geometric multiplicity can be anywhere between 1 and 3. If it’s 3, we are fine, if it’s less than 3, we’re missing out at least one linearly independent eigenvector. And in such a case the matrix would not be diagonalisable. (Conveying mathematical meaning)

And that’s the big observation that we need to get at next week, that a matrix is diagonalisable if and only if all the geometric multiplicities are equal to the algebraic multiplicities. (Didactic – goals of teaching)
2. Collaborative learning in mathematical modelling

Mathematics Educator teaching mathematics to engineering students in a one-semester second year module.

Use of mathematical modelling tasks by students in small groups (4 to 5 students) as a complement to traditional style lectures.

Modelling tasks designed to address mathematical topics such as ordinary differential equations.

Based socioculturally, with attention to the complexity of (social) factors mediating human activity.

**Research question:**

*How do social interactions in small group collaborative work influence the students’ mathematical sense making and the outcomes of the activity?*

Hernandez-Martinez & Harth, 2015
Data from observations of students’ activity were transcribed and analysed within the CHAT frame with close attention to interactions between the students in a group.

Interactions determine the tools available to the group, which in turn mediate the sense making process and influence the outcome of the activity.

Key elements are:
• The community (with their members’ individual histories of previous and present engagement with mathematics),
• The rules (explicit and implicit) and
• The division of labour (which influences whose ideas are valuable or not)
Learning from the research

Analyses showed that students had difficulties with engaging in meaningful mathematical conversation and thinking within a group related to the wider social context of university mathematics teaching. It raises issues for teaching related to preparing students for the needs and expectations of group work that is designed for their deeper mathematical understandings.
Outcomes of analysis

Raises issues for teaching related to preparing students for the needs and expectations of group work that is designed for their deeper mathematical understandings.

Students

difficulties with engaging in meaningful mathematical conversation and thinking within a group

embedded in

the wider social context of university mathematics teaching

Auckland 2016
3. Second Year Mathematics Beyond Lectures
(SYMBoL -- HESTEM Project) + peer support

Curriculum development (summer) project

Two 2nd year mathematics modules (Vector Spaces and Complex Variables) with experienced mathematicians as lecturers

Modules known to be found ‘hard’ by students and success rate was low.

4 interns (end-2nd year maths undergraduates) employed for 6 weeks.

Aim was to get students’ perspectives on what might be provided to help raise achievement.

Interns worked with lecturers to provide resources for students in the two modules.

Support from large group of mathematics staff
• Interns worked on their resources + a discussion each day over tea with as many of the mathematics staff as were around.

• Discussions rich in mathematics -- students and staff acknowledged learning about mathematics and its learning/teaching;

• Both groups felt that growth of mutual understandings were important to staff-student relations in the department.

• Data, collected and analysed throughout the 6 weeks, contributed to a doctoral study.
  – Observations
  – Interviews
  – Surveys

In the following academic year, each module was taught (by the same lecturers) using the material the students had designed.

Peer Support →
Outcomes from SYMBoL – Peer Support

• Important learning by both interns and mathematics staff
• Designed resources used in modules with future cohorts, and/or in peer-supported tutorials
• Creation of a peer support system -- third year students held (voluntary) tutorials each week with the second year students taking the two modules
• Peer leaders ‘trained’ by staff in the Mathematics Education Centre and University Teaching Centre to enact a student-centred pedagogy.
• Tutorials well-received by second-year students (different learning culture) and so continued into a second year.

• Second year students who participated in these tutorials had a higher achievement in their final examinations, even after controlling for their lecture attendance and prior attainment (Duah, Croft & Inglis, 2013).

• Data were collected throughout the peer support activity, and analysed. A thesis documenting the SYMBoL study + Peer Support is forthcoming (Duah, forthcoming).
What we learn …

Relationships between mathematics, learning and teaching
• How teachers think about their teaching
• How teaching relates to students’ learning
• Insights into students’ meaning making in mathematics
• Mathematicians and Mathematics Educators collaborating for students’ improved learning of mathematics
Thank You

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References


