Loughborough University

Mathematics Education Centre

Teaching Mathematics for Student Understanding

Teaching Mathematics at University Level

- Considerable research over 30 years into teaching mathematics at school level (PME 1990)
- Research into teaching at university level less well developed
  - Professional literature
  - Pedagogical literature
  - Research literature

What can we learn from the literature? –Treffert-Thomas & Jaworski, 2015

Scarcity of research on teaching practice

- … while some mathematicians have written about their teaching,
- others have analyzed aspects of their teaching and their students' learning in innovative collegiate courses,
- and a diverse body of other scholarship mentions collegiate mathematics teaching,
- very little research has focused directly on teaching practice—what teachers do and think daily, in class and out, as they perform their teaching work.

(Speer, Smith and Horvath, 2010, p. 99).

Developmental research

Developmental research is research which has the intention not only to chart, monitor, or evaluate the developmental process, but also to contribute to that development (Jaworski, 2003).

> [It is] research which both studies the developmental process and, simultaneously, promotes development through engagement and questioning. …
>
> (Jaworski & Goodchild, 2006, p. 353).

MEC Projects – 3 examples

1. The Linear Algebra Project
2. Collaborative learning in Mathematical Modelling
3. Second-Year Mathematics Beyond Lectures (The SYMBoL Project)

1. The Linear Algebra Project

Focus on lecturer's teaching approach

- A collaborative study between 2 mathematics educator-researchers and one mathematician-lecturer teaching a first year module in linear algebra
- Intensive discussion/reflection and lecture observation over one semester
- Focus on lecturer's actions and goals and their relation to student learning

Jaworski, Treffert-Thomas & Bartsch (2009)

Treffert-Thomas (PhD thesis) (2012)

Collaborative research

Co-learning inquiry: small community of inquiry

- Trusting relationship – bringing mathematics and mathematics education closer together
- Mathematician reflecting using language of linear algebra and his goals in teaching LA
- Mathematics educators tentatively introducing educational terms (e.g., enculturation)
- Developing comfortable ways of speaking to each other.
- Some data from students.

Expository and didactic modes of reflection

Lecturer's words in a research meeting:

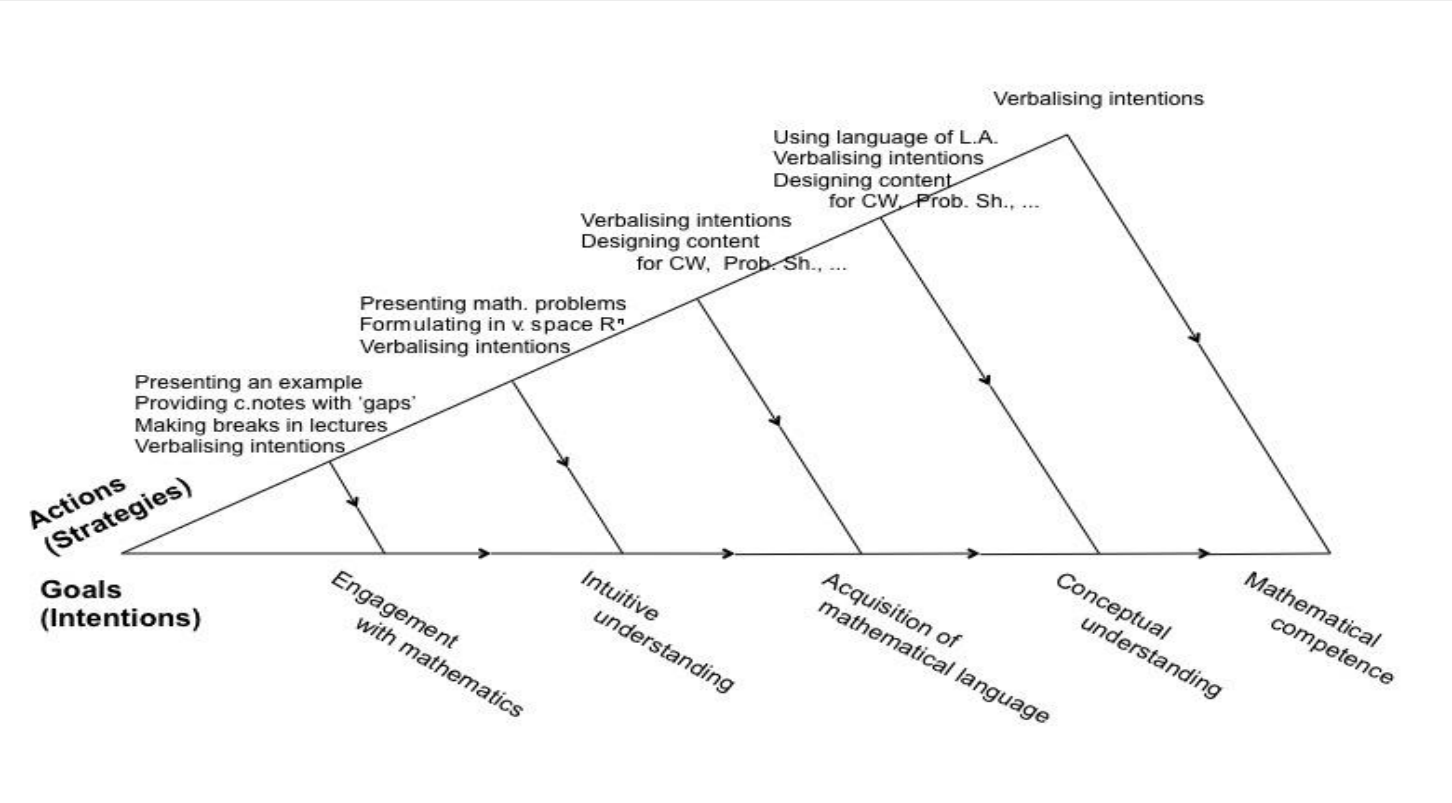
Thursday is about defining the characteristic polynomial, understanding that its zeroes are the eigenvalues, and I'll show an example of an eigenvalue that has algebraic and geometric multiplicity 2. (Didactic – actions of teaching)

Algebraic multiplicity, meaning this is the power with which the factor lamda minus eigenvalue appears in the characteristic polynomial, and geometric multiplicity is the number of linearly independent eigenvectors. (Conveying mathematical meaning)

And these are the important concepts for determining if a matrix is diagonalisable because, for that, we need sufficiently many linearly independent eigenvectors. (Didactic – goals of teaching)

Now if an eigenvalue has algebraic multiplicity larger than 1, that means there are correspondingly fewer eigenvalues. So, in principle, we can fail to find as many eigenvectors as we need in that case. On the other hand, if an eigenvector has algebraic multiplicity 3, the geometric multiplicity can be anywhere between 1 and 3. If it's 3, we are fine, if it's less than 3, we're missing out at least one linearly independent eigenvector. And in such a case the matrix would not be diagonalisable. (Conveying mathematical meaning)

And that's the big observation that we need to get at next week, that a matrix is diagonalisable if and only if all the geometric multiplicities are equal to the algebraic multiplicities. (Didactic – goals of teaching)


Verbalising intentions
Using language of L.A.
Verbalising intentions
Designing content
for CW, Prob. Sh., ...
Verbalising intentions
Designing content
for CW, Prob. Sh., ...
Presenting math. problems
Formulating in v. space R"
Verbalising intentions
Presenting an example
Providing c.notes with 'gaps'
Making breaks in lectures
Verbalising intentions
Actions
(Strategies)
Goals
(Intentions)
Engagement with mathematics
Intuitive understanding
Acquisition of mathematical language
Conceptual understanding
Mathematical competence

2. Collaborative learning in mathematical modelling

Mathematics Educator teaching mathematics to engineering students in a one-semester second year module.

Use of mathematical modelling tasks by students in small groups (4 to 5 students) as a complement to traditional style lectures.

Modelling tasks designed to address mathematical topics such as ordinary differential equations

Based socioculturally, with attention to the complexity of (social) factors mediating human activity

Research question:

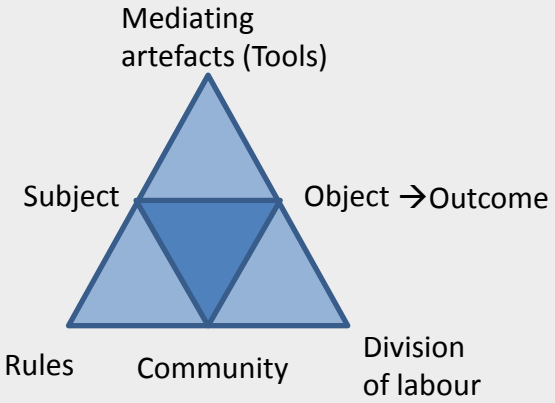
How do social interactions in small group collaborative work influence the students' mathematical sense making and the outcomes of the activity?

Hernandez-Martinez & Harth, 2015

Data from observations of students' activity were transcribed and analysed within the CHAT frame with close attention to interactions between the students in a group.

Interactions determine the tools available to the group, which in turn mediate the sense making process and influence the outcome of the activity.

Cultural-Historical Activity Theory Frame



Key elements are:

- The community (with their members' individual histories of previous and present engagement with mathematics),
- The rules (explicit and implicit) and
- The division of labour (which influences whose ideas are valuable or not)

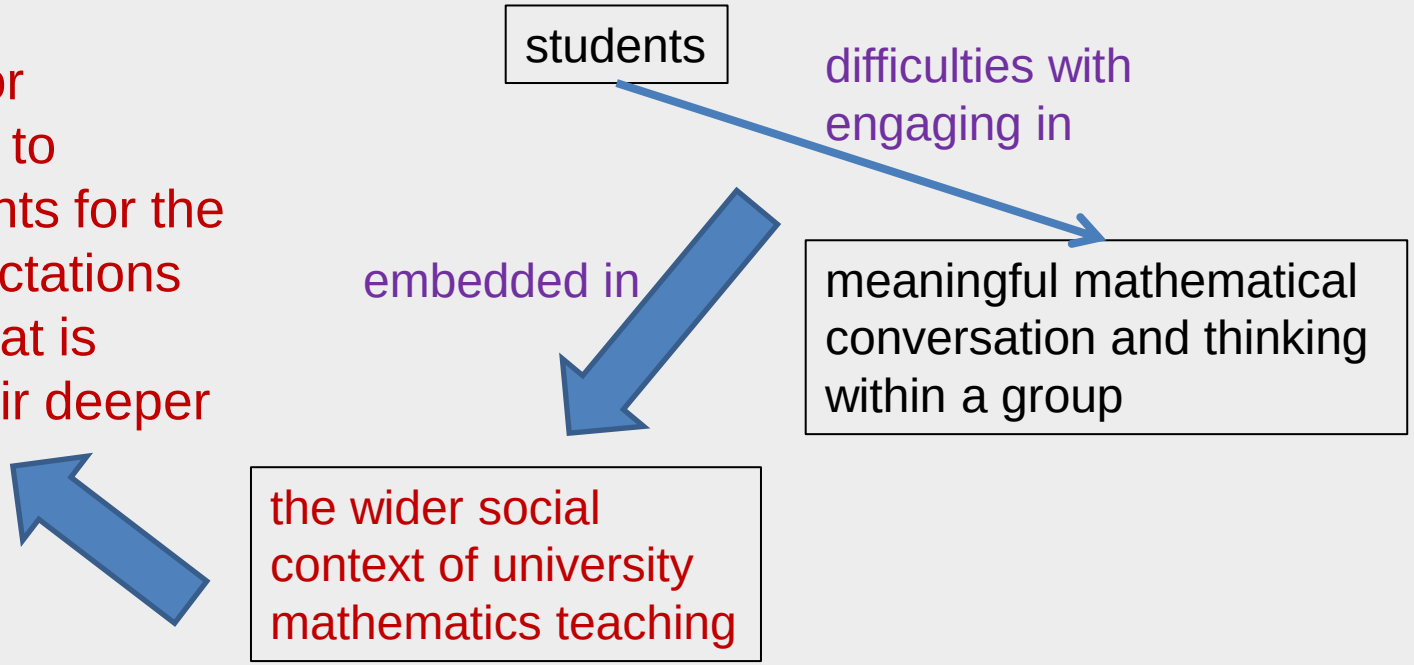
Learning from the research

Analyses showed that students had difficulties with engaging in meaningful mathematical conversation and thinking within a group related to the wider social context of university mathematics teaching.

It raises issues for teaching related to preparing students for the needs and expectations of group work that is designed for their deeper mathematical understandings.

Outcomes of analysis

Raises issues for teaching related to preparing students for the needs and expectations of group work that is designed for their deeper mathematical understandings.



3. Second Year Mathematics Beyond Lectures

(SYMBoL -- HESTEM Project) + peer support

Curriculum development (summer) project

Two 2nd year mathematics modules (Vector Spaces and Complex Variables) with experienced mathematicians as lecturers

Modules known to be found ‘hard’ by students and success rate was low.

4 interns (end-2nd year maths undergraduates) employed for 6 weeks.

Aim was to get students’ perspectives on what might be provided to help raise achievement.

Interns worked with lecturers to provide resources for students in the two modules.

Support from large group of mathematics staff

- Interns worked on their resources + a discussion each day over tea with as many of the mathematics staff as were around.
- Discussions rich in mathematics -- students and staff acknowledged learning about mathematics and its learning/teaching;
- Both groups felt that growth of mutual understandings were important to staff-student relations in the department.

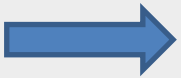
- Data, collected and analysed throughout the 6 weeks, contributed to a doctoral study.
  - Observations
  - Interviews
  - Surveys

In the following academic year, each module was taught (by the same lecturers) using the material the students had designed.

Peer Support →

Outcomes from SYMBoL – Peer Support

- Important learning by both interns and mathematics staff
- Designed resources used in modules with future cohorts, and/or in peer-supported tutorials
- Creation of a peer support system -- third year students held (voluntary) tutorials each week with the second year students taking the two modules
- Peer leaders ‘trained’ by staff in the Mathematics Education Centre and University Teaching Centre to enact a student-centred pedagogy.



- Tutorials well-received by second-year students (different learning culture) and so continued into a second year.
- Second year students who participated in these tutorials had a higher achievement in their final examinations, even after controlling for their lecture attendance and prior attainment

(Duah, Croft & Inglis, 2013).

- Data were collected throughout the peer support activity, and analysed. A thesis documenting the SYMBoL study + Peer Support is forthcoming

(Duah, forthcoming)

What we learn …

Relationships between mathematics, learning and teaching

- How teachers think about their teaching
- How teaching relates to students' learning
- Insights into students' meaning making in mathematics
- Mathematicians and Mathematics Educators collaborating for students' improved learning of mathematics

Thank You

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References

Abdulwahed, M., Jaworski, B. & Crawford, A. R. (2012). Innovative approaches to teaching mathematics in higher education: a review and critique. Nordic Studies in Mathematics, Education, 17 (2), 101-120.

Alpers, B. (2008). The mathematical expertise of mechanical engineers - The case of machine element dimensioningin: Alpers, B. et al. (eds.): Mathematical Education of Engineers, Proc. of 14th SEFI (MWG) Conference joint with IMA, Loughborough

Black, L., Williams, J., Hernandez-Martinez, P., Davis, P., Pampaka, M., Wake, G. (2010). Developing a 'leading identity': the relationship between students' mathematical identities and their career and higher education aspirations. Educational Studies in Mathematics Volume 73, Issue 1, pp 55-72

Burn, R.P. (1982) *A Pathway into Number Theory*. Cambridge, UK: Cambridge University Press.

Burn, B, Appelby, J. & Maher, P. (1998). *Teaching undergraduate mathematics.* London: Imperial College Press.

Chang, J. M. (2011). A practical approach to inquiry-based learning in linear algebra. *International Journal of Mathematical Education in Science and Technology,* 42 (2), 245–259.

Duah, F., Croft, T., & Inglis, M. (2014). Can peer assisted learning be effective in undergraduate mathematics?. *International Journal of Mathematical Education in Science and Technology*, *45*(4), 552-565.

Duah, F. K. (in press). Staff-student Collaboration in Advanced Undergraduate Mathematics Course Design and Delivery. Unpublished PhD Thesis, Loughborough University, UK.

Dubinsky, E. (1987). Teaching mathematical induction I. *Journal of Mathematical Behavior*, **6**(1), 305-317,

Felder, R. (1993) 'Reaching the second tier: Learning and teaching styles in college science education', *Journal of College Science Teaching*, 23(5), pp286-290.

Hawkes, T., & Savage, M., (Eds.) (2000). *Measuring the Mathematics Problem.* London, UK: The Engineering Council.

Hemmi, K. (2010). **Three styles characterising mathematicians' pedagogical perspectives on proof** Educational Studies in Mathematics. Volume 75, Issue 3, pp 271-291

Hernandez-Martinez, P. and Harth, H. (2015) An Activity Theory analysis of group work in mathematical modelling. In K. Beswick, T. Muir & J. Wells (Eds.) Proceedings of the 39th Psychology of Mathematics Education conference, vol. 3 (pp. 57 – 64), Hobart, Australia: PME.

Holton D. (Ed.) (2001). *Teaching and learning of mathematics at university level: An ICMI study.* The Netherlands: Kluwer Academic Publishers.

Jaworski, B. (2002). Sensitivity and Challenge in University Mathematics Teaching. Educational Studies in Mathematics, 51, 71-94.

Jaworski, B. (2003) Research practice into/influencing mathematics teaching and learning development: towards a theoretical framework based on co-learning partnerships. *Educational Studies in Mathematics* 54, 2-3, 249-282.

Jaworski, B., Mali, A., & Petropoulou, G. (in review). Making sense of undergraduate mathematics teaching for students' meaning-making in mathematics.

Jaworski, B. & Matthews, J. (2011) Developing teaching of mathematics to first year engineering students. *Teaching Mathematics and Its Applications* 30(4): 178-185.

Jaworski B, Robinson C, Matthews J, Croft A.C. (2012). An activity theory analysis of teaching goals versus student epistemological positions. *International Journal of Technology in Mathematics Education*, *19*(4), 147-152.

Jaworski, B. & Potari, D. (2009). *Bridging the macro-micro divide*: using an activity theory model to capture complexity in mathematics teaching and its development. *Educational Studies in Mathematics*, *72*, 219-236.

Jones, M. (2008). Pedagogical literature: What can be learned and where to begin? *Primus*, 18 (3), 291–298.

Mason, J. (2002) *Mathematics Teaching Practice: A Guide for University and College Lecturers*. Cambridge, UK: Horwood Publishing Limited.

Marton F. & Säljö R. (1976) "On Qualitative Differences in Learning — 1: Outcome and Process" *Brit. J. Educ. Psych.* 46, 4-11

Millet, K. (2001). Making large lectures effective: an effort to increase student success. In D. Holton (Ed.).

Mokhtar, M. Z. & Tarmizi, R. A. (2010). Enhancing calculus learning engineering students through problem-based learning. *WSEAS transactions on advances in engineering education*, 7 (8), 255–264.

Nardi, E. (2008). Amongst Mathematicians: Teaching and learning mathematics at university level. London: Springer.

O'Callaghan, B.R. (1998). Computer-intensive algebra and students' conceptual knowledge of functions. Journal for Research in Mathematics Education, 29(1), 21-40.

Padraig, M. & McLoughlin, M. (2010) 'Aspects of a neoteric approach to advance students' ability to conjecture, prove, or disprove'. Annual Summer Meeting of the Mathematical Association of America. Pittsburgh, Pennsylvania, August 2010.

Potari, D. & Jaworski, B. (2002) Tackling Complexity in Mathematics Teacher Development: Using the teaching triad as a tool for reflection and enquiry. *Journal of Mathematics Teacher Education*, *5*(4), 351-380.

Pritchard, P. (2015). Lectures and Transition: from bottles to bonfires. In, M. Grove, T. Croft, J. Kyle, and D Lawson, Transitions in Undergraduate Mathematics. Birmingham: University of Birmingham; The Higher Education Academy.

Rasmussen, C & Kwon O. N. (2007). An inquiry-oriented approach to undergraduate mathematics. Journal of Mathematical Behavior 26 (2007) 189–194

Richardson, V. (2003). Constructivist pedagogy. *The Teachers College Record*, 105 (9), 1623–1640.

Ridgeway, J., Swan, M. & Burkhardt, H. (2001). Assessing mathematical thinking via FLAG. In D. Holton (Ed.).

Rodd, M. and M. Brown (2005). Hardly Hardy: vulnerability and undergraduate mathematics students' identities. Kingfisher DELTA05, Queensland, Australia, Queensland University.

Rowland, T., Huckstep, P., & Thwaites, A. (2005). Elementary teachers' mathematics subject knowledge: The knowledge quartet and the case of Naomi. Journal of Mathematics Teacher Education, 8, 255–281

Rowland, T. (2009). Beliefs and actions in university mathematics teaching. Proceedings of PME 33 vol 5, 17-24,

Solomon, Y. (2007) 'Not belonging? What makes a functional learner identity in undergraduate mathematics?', *Studies in Higher Education*, 32(1), pp79-96.

Speer, N. M., Smith III, J. P., & Horvath, A. (2010). Collegiate mathematics teaching: An unexamined practice. *The Journal of Mathematical Behavior*, 29, 99-114.

Thomas, S. (2012). *An activity theory analysis of linear algebra teaching within university mathematics*. Unpublished doctoral dissertation. Loughborough University, UK.

Treffert-Thomas, S. (2015). Conceptualising a university teaching practice in an activity theory perspective. Nordic Studies in Mathematics Education, 20,2, 53-78.

Treffert-Thomas, S., & Jaworski, B. (2015). Developing mathematics teaching: what can we learn from the literature? In M. Grove, T. Croft, J. Kyle, & D. Lawson (Eds.), *Transitions in Undergraduate Mathematics, pp 259-276*. Birmingham: University of Birmingham with Higher Education Academy.

Uhlig, F. (2003) 'A new unified, balanced, and conceptual approach to teaching linear algebra', *Linear Algebra and Its Applications*, 361, pp147-159.

Williams, J. (2012). "Use and exchange value in mathematics education: contemporary CHAT meets Bourdieu's sociology." *Educational Studies in Mathematics* 80, no. 1-2(2012) : 57-72.

Wu, H. (1999). The joy of lecturing. In S. G. Krantz (Ed.). How to teach mathematics. The American Mathematical Society, pp. 261-271.

Yackel, E. & Cobb, P. (1996). Sociomathematical Norms, Argumentation, and Autonomy in Mathematics. *Journal for Research in Mathematics Education* Vol. 27, No. 4 pp. 458-